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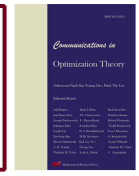

PERMANENCE IN A NONAUTONOMOUS ONE PREDATOR-TWO COOPERATIVE PREY SYSTEMS WITH DELAYS

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Abstract. In this paper, we consider a class of non-autonomous three species ratio-dependent one predator-two cooperative prey systems with pure discrete time delays and establish sufficient conditions which ensure the system to be permanent.

Keywords. Predator-prey-cooperative system; Permanence; Ratio-dependent; Discrete time delay.

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1. Introduction

It is known that the Lotka-Volterra population dynamical system is one of important disciplines in modern applied mathematics, where population dynamical competitive systems, population dynamical cooperative systems, population dynamical predator-prey systems are become the most popular topics among the scholars. There have been a lot of studies related to the population Lotka-Volterra dynamical systems [1, 2, 3, 4, 5, 6, 7, 8] and the references cited therein. Especially, the LotkaCVolterra competitive system, predator-prey system and cooperative system characterize competitive-cooperative [1, 2, 3] and competitive-predator-prey [4, 5] interactions between species that are of great interest in the study of dynamical behaviors of Lotka-Volterra systems. Most of these studies concerned with the extinction, permanence, global attractivity and the existence of periodic solution and so on. For example, in [1], Lv, Yan and Lu considered the following competitor-competitor-cooperative Lotka-Volterra systems with pure delays

$$\begin{aligned}
\dot{x}_1(t) = \;& x_1(t)\big(r_1(t) - a_{11}(t)x_1(t-\tau_{11}(t)) \\
& -a_{12}(t)x_2(t-\tau_{12}(t)) + a_{13}(t)x_3(t-\tau_{13}(t))\big), \\
\dot{x}_2(t) = \;& x_2(t)\big(r_2(t) - a_{21}(t)x_1(t-\tau_{21}(t)) \\
& -a_{22}(t)x_2(t-\tau_{22}(t)) + a_{23}(t)x_3(t-\tau_{23}(t))\big), \\
\dot{x}_3(t) = \;& x_3(t)\big(r_3(t) + a_{31}(t)x_1(t-\tau_{31}(t)) \\
& +a_{32}(t)x_2(t-\tau_{32}(t)) - a_{33}(t)x_3(t-\tau_{33}(t))\big).
\end{aligned} \tag{1.1}$$

They obtained sufficient conditions for the existence and global attractivity of periodic solutions by Krasnosselsii's fixed point theorem and the construction of Liapunov functions for system (1.1). Base on the

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above works, Muhammadhaji, Teng and Rehim [2] studied the following three species non-autonomous Lotka-Volterra competitive-cooperative systems with delays

$$\begin{aligned} \dot{x}_1(t) = &\ x_1(t)\big[r_1(t) - a_{11}^1(t)x_1(t-\tau) - a_{11}^2(t)x_1(t-2\tau) \\ &-a_{12}(t)x_2(t-2\tau) + a_{13}(t)x_3(t-\tau)\big], \\ \dot{x}_2(t) = &\ x_2(t)\big[r_2(t) - a_{21}(t)x_1(t-2\tau) - a_{22}^1(t)x_2(t-\tau) \\ &-a_{22}^2(t)x_2(t-2\tau) + a_{23}(t)x_3(t-\tau)\big], \\ \dot{x}_3(t) = &\ x_3(t)\big[r_3(t) + a_{31}(t)x_1(t-\tau) + a_{32}(t)x_2(t-\tau) \\ &-a_{33}^1(t)x_3(t) - a_{33}^2(t)x_3(t-\tau)\big]. \end{aligned} \tag{1.2}$$

They obtained some sufficient conditions on the permanence of species and the global attractivity of the system are established by construction of Liapunov functional and the comparison method given in [6]. In [4], Muhammadhaji and Teng considered the following periodic ratio-dependent competing predatorCprey system with stage structure

$$\begin{aligned} \dot{x_1}(t) = &\ r(t)x_2(t) - B(t)x_1(t) - d_1(t)x_1^2(t), \\ \dot{x_2}(t) = &\ B(t)x_1(t) - d_2(t)x_2^2(t) - \frac{a_1(t)x_2(t)y_1(t)}{k(t)x_2^2(t) + \beta_1(t)x_2(t) + \alpha_1(t)} \\ &- \frac{a_2(t)x_2(t)y_2(t)}{\alpha_2(t) + \beta_2(t)x_2(t) + \gamma(t)y_2(t)}, \\ \dot{y_1}(t) = &\ y_1(t)\big(-d_3(t) + \frac{e_1(t)x_2(t)}{k(t)x_2^2(t) + \beta_1(t)x_2(t) + \alpha_1(t)} - D_1(t)y_1(t) - \frac{c_1(t)y_2(t)}{b_1(t) + y_2(t)}\big), \\ \dot{y_2}(t) = &\ y_2(t)\big(-d_4(t) + \frac{e_2(t)x_2(t)}{\alpha_2(t) + \beta_2(t)x_2(t) + \gamma(t)y_2(t)} - D_2(t)y_2(t) - \frac{c_2(t)y_1(t)}{b_2(t) + y_1(t)}\big). \end{aligned} \tag{1.3}$$

They obtained some sufficient conditions on the permanence, extinction and periodic solution of a competing periodic predator-prey system with functional response and stage structure by means of comparison theorem.

However, characterize predator-prey and cooperative interactions between species are fairly rare. For that reason and based on the above works, in the present paper, we consider the following three species non-autonomous ratio-dependent Lotka-Volterra type one predator-two cooperative prey systems with pure discrete time delays

$$\begin{aligned} \dot{x}_1(t) = &\ x_1(t)\big[r_1(t) - a_1(t)x_1(t-\tau_1) - \frac{b_1(t)x_3(t-\tau_1)}{c_1(t)x_2(t-\tau_1) + n_1(t)x_3(t-\tau_1)}\big], \\ \dot{x}_2(t) = &\ x_2(t)\big[r_2(t) - a_2(t)x_2(t-\tau_2) - \frac{b_2(t)x_3(t-\tau_2)}{c_2(t)x_1(t-\tau_2) + n_2(t)x_3(t-\tau_2)}\big], \\ \dot{x}_3(t) = &\ x_3(t)\big[-r_3(t) + \frac{b_3(t)x_1(t-\tau_3)}{c_3(t)x_1(t-\tau_3) + n_1(t)x_3(t-\tau_3)} \\ &+ \frac{b_4(t)x_2(t-\tau_3)}{c_4(t)x_2(t-\tau_3) + n_2(t)x_3(t-\tau_3)} - a_3(t)x_3(t-\tau_3)\big]. \end{aligned} \tag{1.4}$$

Our main purpose in this paper is to establish some sufficient conditions on the boundedness and permanence for system (1.4). The method used in this paper is the comparison method given in [6].

2. Preliminaries

In system (1.4), we have that $x_i(t)(i=1,2)$ denote the density of the two cooperative prey species at time t, respectively; $x_3(t)$ denotes the density of the predator species at time t. $r_i(t)(i=1,2,3)$ denote the intrinsic growth rate of species $x_i(i=1,2,3)$ at time t, respectively; $a_i(t)(i=1,2,3)$ denote the intrapatch restriction density of species $x_i(i=1,2,3)$ at time t,respectively; $c_1(t)$ and $c_2(t)$ represent the cooperative coefficients between two species x_1 and x_2 at time t,respectively; $b_1(t)$ and $b_2(t)$ are the capturing rate of the predator species at time t, $b_3(t)/b_1(t)$ and $b_4(t)/b_2(t)$ are the rate of conversion of nutrients into the reproduction of the two predators at time t, respectively. Throughout this paper, we always assume that system (1.4) satisfies the following assumption

(H_1) $\tau_i>0(i=1,2,3)$ is a positive constant, $r_i(t),a_i(t)(i=1,2,3)$, $b_i(t)(i=1,2,3,4)$, $c_i(t)(i=1,2,3,4)$ and $n_i(t)(i=1,2)$ are continuous, bounded and strictly positive functions on $[0,\infty)$.

Due to the biological meaning of the model, the initial conditions for system (1.4) take the form

$$x_i(t)=\phi_i(t) \quad \text{for all} \quad t\in[-\tau,0],\ i=1,2,3, \tag{2.1}$$

where $\phi_i(t)$ $(i=1,2,3)$ are nonnegative continuous functions defined on $[-\tau,0)$ satisfying $\phi_i(0)>0$ $(i=1,2,3)$ and $\tau=\tau_1+\tau_2+\tau_3$.

Throughout this paper, for a continuous function $f(t)$, we set

$$f^M=\max_{t\in[0,\infty)} f(t), \quad f^L=\min_{t\in[0,\infty)} f(t).$$

Now, we present some useful definition and lemmas.

Definition 2.1. System (1.4) is said to be permanent if there exist positive constants m,M and T_0, such that each positive solution $(x_1(t),x_2(t),x_3(t))$ of system (1.4) with any positive initial value φ, fulfill $m\le x_i(t)\le M(i=1,2,3)$ for all $t\ge T_0$, where T_0 may depend on φ.

Lemma 2.2. [6] *Assume that function $y(t)\ge 0$ defined on $[-m\tau,\infty)$ satisfies that*

$$\dot y(t)\le y(t)(\lambda-\sum_{l=1}^{m}\mu^l y(t-l\tau))+D,$$

$$\dot y(t)\le y(t)(\lambda-\sum_{l=1}^{m}\mu^l y(t-l\tau))+D,$$

where

$$\lambda>0,\ \mu^l\ge 0(l=0,1,2,\cdots m),\ \mu=\sum_{l=0}^{m}\mu^l>0,\ D\ge 0,$$

are constants. Then there exists a positive constant M_y such that

$$\limsup_{t\to\infty} y(t)\le M_y=-\frac{D}{\lambda}+(\frac{D}{\lambda}+y^*)\exp(\lambda m\tau), \tag{2.2}$$

where $y=y^*$ is the unique positive solution of equation

$$y(\lambda-\mu y)+D=0.$$

Lemma 2.3. [6] *Assume that function $y(t)\ge 0$ defined on $[-m\tau,\infty)$ satisfies that*

$$\dot y(t)\ge y(t)(\lambda-\sum_{l=1}^{m}\mu^l y(t-l\tau))+D,$$

where

$$\lambda > 0,\ \mu^l \geq 0 (l = 0,1,2,\cdots m),\ \mu = \sum_{l=0}^{m} \mu^l > 0 \quad and\ D \geq 0,$$

are constants. If (2.2) holds, then there exists a positive constant m_y such that

$$\liminf_{t\to\infty} y(t) \geq m_y = \frac{\lambda}{\mu} \exp\{(\lambda - \mu M_y) m\tau\}.$$

Lemma 2.4. [7] *Consider the following equation:*
$\dot{u}(t) = u(t)(d_1 - d_2 u(t)),$
where $d_2 > 0$. Then
$(1) If\, d_1 > 0, then \lim_{t\to+\infty} u(t) = d_1/d_2.$
$(2) If\, d_1 < 0, then \lim_{t\to+\infty} u(t) = 0.$

Lemma 2.5. [8] *If there exist positive constants m and M for any $\Phi \in C_+^n[-\tau, 0]$ such that*

$$m < \liminf_{t\to\infty} x_i(t,0,\Phi) \leq \limsup_{t\to\infty} x_i(t,0,\Phi) < M, \qquad i = 1,2,\cdots,n,$$

then the following periodic general functional differential equation

$$\frac{dx}{dt} = F(t, x_t)$$

admits at least one positive $\omega-$periodic solution, where $x(t) \in R^n$ and $F(t,x_t)$ is a $n-$dimensional continuous functional, $x(t,0,\Phi) = (x_1(t,0,\Phi), x_2(t,0,\Phi), \cdots, x_n(t,0,\Phi))$ is a solution of the functional differential equation with initial condition $x_0 = \Phi$.

3. Main results

In this section, we will obtain some sufficient conditions for the boundedness and permanence of system (1.4).

Theorem 3.1. *Assume that (H_1) holds, then system (1.4) is ultimately bounded.*

Proof. First, we show that $x_1(t)$ is ultimately bounded. From the first equation of system (1.4), we have

$$\dot{x_1}(t) \leq x_1(t)\big(r_1^M - a_1^L x_1(t-\tau_1)\big),$$

By Lemma 2.2, we get

$$\limsup_{t\to\infty} x_1(t) \leq M_1 \triangleq \frac{r_1^M}{a_1^L} \exp\big(r_1^M \tau_1\big).$$

For $x_2(t)$, we can obtain

$$\limsup_{t\to\infty} x_2(t) \leq M_2 \triangleq \frac{r_2^M}{a_2^L} \exp\big(r_2^M \tau_2\big).$$

Finally, from third equation of system (1.4), we have

$$\dot{x_3}(t) \leq x_3(t)\big(r_0 - a_3^L x_3(t-\tau_3)\big),$$

where $r_0 = \dfrac{b_3^M}{c_3^L} + \dfrac{b_4^M}{c_4^L}$. Usig Lemma 2.2, we get

$$\limsup_{t\to\infty} x_3(t) \leq M_3 \triangleq \frac{r_0}{a_3^L} \exp\big(r_0 \tau_3\big).$$

This completes the proof. □

Theorem 3.2. *Assume that (H_1) holds and $A_i > 0$ $(i = 1,2)$. Then for any positive solution $(x_1(t), x_2(t), x_3(t))$ of system (1.4), we have*

$$\liminf_{t\to\infty} x_1(t) \geq m_1 = \frac{A_1}{a_1^M} \exp\left\{(A_1 - a_1^M M_1)\tau_1\right\}$$

and

$$\liminf_{t\to\infty} x_2(t) \geq m_2 = \frac{A_2}{a_2^M} \exp\left\{(A_2 - a_2^M M_2)\tau_2\right\},$$

where $A_i = r_i^L - \dfrac{b_i^M}{n_i^L} (i = 1,2)$.

Proof. From the first and second equations of system (1.4), we have

$$\dot{x_1}(t) \geq x_1(t)\big(r_1^L - \frac{b_1^M}{n_1^L} - a_1^M x_1(t-\tau_1)\big),$$

and

$$\dot{x_2}(t) \geq x_2(t)\big(r_2^L - \frac{b_2^M}{n_2^L} - a_2^M x_2(t-\tau_2)\big).$$

Using Lemma 2.3, we get

$$\liminf_{t\to\infty} x_i(t) \geq m_i = \frac{A_i}{a_i^M} \exp\left\{(A_i - a_i^M M_i)\tau_i\right\}, \quad i = 1,2.$$

where $A_i = r_i^L - \dfrac{b_i^M}{n_i^L} (i = 1,2)$. This completes the proof. □

Theorem 3.3. *Assume that the conditions of Theorem 3.2 hold and $A_3 > 0$. Then system (1.4) is permanent, where $A_3 = \dfrac{b_3^L m_1}{c_3^M M_1 + n_1^M M_3} + \dfrac{b_4^L m_2}{c_4^M M_2 + n_2^M M_3} - r_3^M$.*

Proof. From Theorem 3.1 and Theorem 3.2, for any positive constant $\varepsilon_0 > 0$, there exists a positive constant T_0 such that

$$x_i(t) \leq M_i + \varepsilon_0, \quad i = 1,2,3, \;\; x_i(t) \geq m_i + \varepsilon_0, \;\; i = 1,2, \text{ for all } \; t \geq T_0.$$

From third equation of system (1.4), we have

$$\dot{x_3}(t) \geq x_3(t)\big(\frac{b_3^L(m_1+\varepsilon_0)}{c_3^M(M_1+\varepsilon_0) + n_1^M(M_3+\varepsilon_0)} + \frac{b_4^L(m_2+\varepsilon_0)}{c_4^M(M_2+\varepsilon_0) + n_2^M(M_3+\varepsilon_0)} - r_3^M - a_3^M x_3(t-\tau_3)\big),$$

Since ε_0 is arbitrary, we find from Lemma 2.3 that

$$\liminf_{t\to\infty} x_3(t) \geq m_3 = \frac{A_3}{a_3^M} \exp\left\{(A_3 - a_3^M M_3)\tau_3\right\},$$

where $A_3 = \dfrac{b_3^L m_1}{c_3^M M_1 + n_1^M M_3} + \dfrac{b_4^L m_2}{c_4^M M_2 + n_2^M M_3} - r_3^M$. Hence, system (1.4) is permanent. This completes the proof. □

From Lemma 2.4, we have the following result.

Corollary 3.4. *Assume that (H_1) holds and $A_4 < 0$. Then predator species x_3 in system (1.4) is go to extinction,here $A_4 = \dfrac{b_3^M}{c_3^L} + \dfrac{b_4^M}{c_4^L} - r_3^L$.*

From Lemma 2.5, we have the following result.

Corollary 3.5. *Assume that the conditions of Theorem 3.3 hold. Then system (1.4) admits at least one positive $\omega-$periodic solution.*

4. EXAMPLES

Example 4.1. Consider the following system

$$\begin{aligned}
\dot{x}_1(t) = \ & x_1(t)\big(2-0.5|sin(t)|-(2+|sin(t)|)x_1(t-0.03)\\
&-\frac{(0.1+0.01|cos(t)|)x_3(t-0.03)}{(0.45+|sin(t)|)x_2(t-0.03)+(0.15+0.1|cos(t)|)x_3(t-0.03)}\big),\\
\dot{x}_2(t) = \ & x_2(t)\big(2-0.4|sin(t)|-(2+|sin(t)|)x_2(t-0.02)\\
&-\frac{(0.1+0.01|cos(t)|)x_3(t-0.02)}{(0.45+|sin(t)|)x_1(t-0.02)+(0.15+0.1|cos(t)|)x_3(t-0.02)}\big),\\
\dot{x}_3(t) = \ & x_3(t)\big(-0.05-0.05|sin(t)|-(4.5+0.1|sin(t)|)x_3(t-0.01)\\
&+\frac{(1+|cos(t)|)x_1(t-0.01)}{(1+0.1|sin(t)|)x_1(t-0.01)+(0.15+0.1|cos(t)|)x_3(t-0.01)}\\
&+\frac{(1+|cos(t)|)x_2(t-0.01)}{(1+0.1|sin(t)|)x_2(t-0.01)+(0.15+0.1|cos(t)|)x_3(t-0.01)}\big).
\end{aligned} \tag{4.1}$$

By directly calculation, we can get

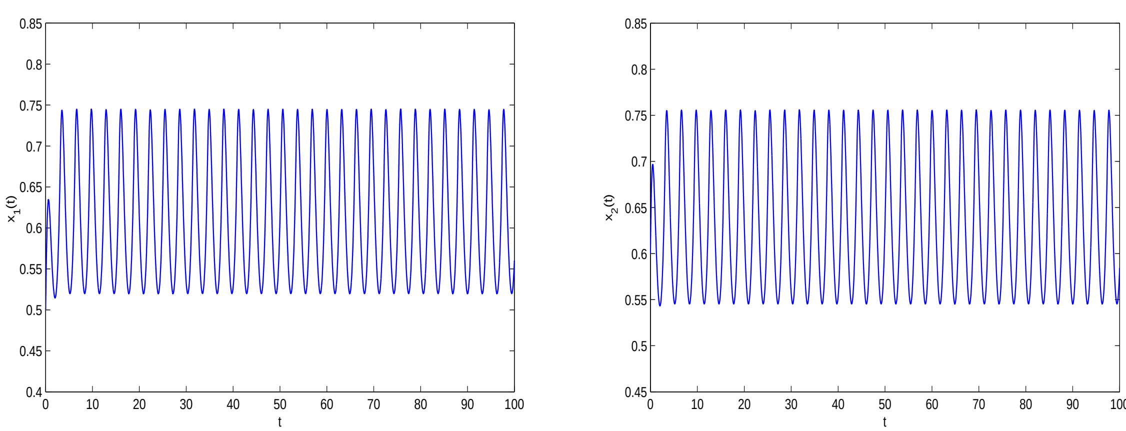
$$A_1 = (2-0.5|sin(t)|)^L - \frac{(0.1+0.01|cos(t)|)^M}{(0.15+0.1|cos(t)|)^L} \approx 0.7667,$$

$$A_2 = (2-0.4|sin(t)|)^L - \frac{(0.1+0.01|cos(t)|)^M}{(0.15+0.1|cos(t)|)^L} \approx 0.8667,$$

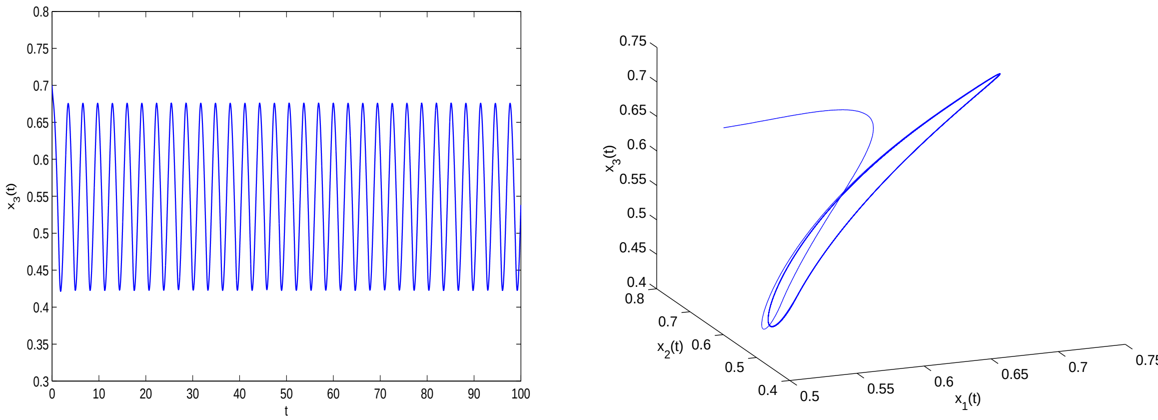
$$M_1 \approx 1.0618, \quad M_2 \approx 1.0408, \quad M_3 \approx 0.9252, \quad m_1 \approx 0.2377, \quad m_2 \approx 0.2761,$$

$$A_3 = \frac{b_3^L m_1}{c_3^M M_1 + n_1^M M_3} + \frac{b_4^L m_2}{c_4^M M_2 + n_2^M M_3} - r_3^M \approx 0.2705$$

It is clear that the conditions of Theorem 3.3 and Corollary 3.5 hold.



(a) (b)



(c) (d)

Fig. 4.1. Dynamics of system (4.1)

From the Fig. 4.1. we can see, system (4.1) is permanent and has a periodic solution.

Example 4.2. Consider the following system

$$\begin{aligned}
\dot{x}_1(t) = \ & x_1(t)\big(2-0.5|sin(t)|-(2+|sin(t)|)x_1(t-0.03) \\
& -\frac{(0.1+0.01|cos(t)|)x_3(t-0.03)}{(0.45+|sin(t)|)x_2(t-0.03)+(0.15+0.1|cos(t)|)x_3(t-0.03)}\big), \\
\dot{x}_2(t) = \ & x_2(t)\big(2-0.4|sin(t)|-(2+|sin(t)|)x_2(t-0.02) \\
& -\frac{(0.1+0.01|cos(t)|)x_3(t-0.02)}{(0.45+|sin(t)|)x_1(t-0.02)+(0.15+0.1|cos(t)|)x_3(t-0.02)}\big), \\
\dot{x}_3(t) = \ & x_3(t)\big(-1.5-|sin(t)|-(3.5+0.1|sin(t)|)x_3(t-0.01) \\
& +\frac{(0.25+|cos(t)|)x_1(t-0.01)}{(2.5+0.5|sin(t)|)x_1(t-0.01)+(0.15+0.1|cos(t)|)x_3(t-0.01)} \\
& +\frac{(0.25+|cos(t)|)x_2(t-0.01)}{(1.5+0.5|sin(t)|)x_2(t-0.01)+(0.15+0.1|cos(t)|)x_3(t-0.01)}\big).
\end{aligned} \tag{4.2}$$

By directly calculation we can get

$$A_1 = (2-0.5|sin(t)|)^L - \frac{(0.1+0.01|cos(t)|)^M}{(0.15+0.1|cos(t)|)^L} \approx 0.7667,$$

$$A_2 = (2-0.4|sin(t)|)^L - \frac{(0.1+0.01|cos(t)|)^M}{(0.15+0.1|cos(t)|)^L} \approx 0.8667,$$

$$A_4 = \frac{b_3^M}{c_3^L} + \frac{b_4^M}{c_4^L} - r_3^L \approx -0.1667 < 0.$$

It is clear that the conditions of Theorem 3.2 and Corollary 3.4 hold.

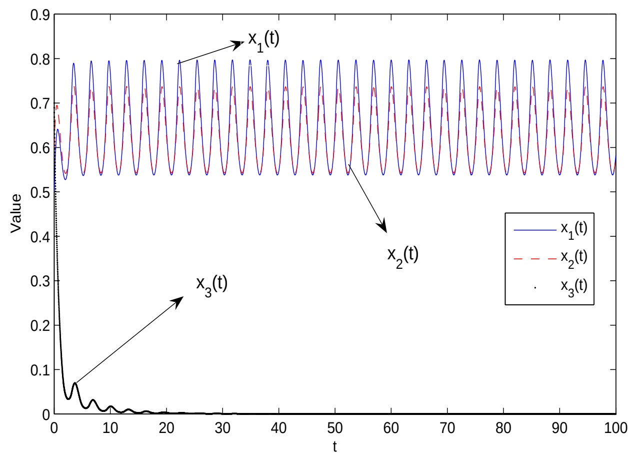


Fig. 4.2. Dynamics of system (4.2)

From the Fig. 4.2. we can see, prey species x_1 and x_2 in system (4.2) are permanent and predator species x_3 in system (4.2) is go to extinction.

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References

[1] X. Lv, P. Yan, S. Lu, Existence and global atrractivity of positive periodic solution of competitor- competitor-mutualist Lotka-Volterra system with deviating arguments, Math. Comput. Model. 51 (2010), 823-832.

[2] A. Muhammadhaji, Z. Teng, M. Rehim, Dynamical behavior for a class of delayed competitiveCmutualism systems, Differ. Equ. Dyn. Syst. 23 (2015), 281-301.

[3] M. Gyllenberg, P. Yan, Y. Wang, Limit cycles for the competitor-competitor-mutualist Lotka-Volterra systems. Phys. D. 221(2006), 135-145.

[4] A. Muhammadhaji, Z. Teng, Permanence and extinction analysis for a periodic competing predator-prey system with stage structure, Int. J. Dynam. Control 5 (2017), 858-871.

[5] Tona TV, Hieu NT, Dynamics of species in a model with two predators and one prey, Nonlin. Anal. 74(2011), 4868-4881.

[6] Y. Nakata and Y. Muroya, Permanence for nonautonomous Lotka-Volterra cooperative systems with delays, Nonl. Anal. 11 (2010), 528-534.

[7] Z. Ma, Z. Li, S. Wang, T. Li, F. Zhang, Permanence of a predator-prey system with stage structure and time delay, Appl. Math. Comput. 201 (2008), 65-71.

[8] Z. Teng, L. Chen, The positive periodic slotions in periodic Kolmogorov type systems with delays, Acta. Math. Appl. Sin. 22 (1999), 446-456 (in Chinese).