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A NOTE ON DURATION-SERVICE-TIME DIFFERENTIATED ENERGY SERVICES

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Abstract. Differentiated energy services (DESs) have proven a promising solution to elicit load flexibility for the supply/demand balance in smart grids. This note explores energy services differentiated by duration requirements and service times, building upon existing models such as duration-differentiated energy services, duration-deadline jointly differentiated energy services, and multiple-arrival multiple-deadline differentiated energy services. A necessary and sufficient condition is derived to check the adequacy of a supply for a continuum of loads consuming the considered DESs. Moreover, to verify the economic feasibility of such DESs, we establish, in a forward market, the existence of an efficient competitive equilibrium, where self-interested DES consumers attain the maximum social welfare, even in the absence of a social planner.

Keywords. Demand response; Matrix completion; Market implementation; Competitive equilibrium.

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1. Introduction

The balance between supply and demand is crucial in power systems. Traditionally, the relatively controllable supply side takes almost full responsibility for maintaining this balance. However, as smart grids enable responsive demand management, the demand side is increasingly contributing to this critical balancing act. Moreover, the growing integration of variable renewable energy sources, such as solar and wind power, into power systems is transforming this landscape. Consequently, achieving a harmonious supply-demand balance necessitates innovative approaches and technologies that can elicit and/or enhance load flexibility [14, 15, 16, 25].

A promising solution involves innovative electricity services. Electricity should no longer be treated as a uniform commodity sold at a standard unit price. Instead, differentiated energy services can be developed to cater to varying levels of demand flexibility. For instance, traditional plug-and-play models fail to leverage the flexibility underlying deferrable and interruptible loads. Particularly, for a load requiring constant power for a specified duration, its flexibility

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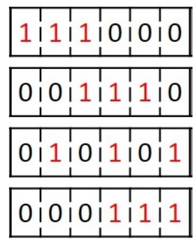

FIGURE 1. A load requiring a service duration of three slots is indifferent to the four different power delivery profiles, where 1 means that constant power is delivered to the load at the corresponding time slot.

lies in that power can be delivered at arbitrary time slots as long as the total duration of power delivery to this load within an operational period is equal to the specified duration, as illustrated in Figure 1. Such loads motivate the study of duration-differentiated energy services, which have been well-studied in [20] and [21]. Moreover, considering loads with varying deadlines for power delivery, duration-deadline jointly differentiated energy services have been proposed and analyzed in [3] and [4]. Furthermore, by relaxing the restrictions on the same arrival time, multiple-arrival multiple-deadline differentiated energy services have been investigated in [12] and [13]. Note that the service time for a given load may not be continuous; in other words, it can be split into multiple non-adjacent intervals rather than occurring in one uninterrupted block. Therefore, it cannot be simply characterized by an arrival time and a deadline. Thus, to accommodate the flexibility of such loads, we explore, in this note, energy services differentiated by duration requirements and general service times. The duration of such a differentiated energy service (DES) specifies the quantity of power to be delivered, while the service time of this DES consists of the time slots available for power delivery; moreover, the total number of time slots for actual power delivery via this DES equals the specified duration. A more flexible load can subscribe to a DES with a shorter duration or a broader service time.

In this note, we derive a necessary and sufficient condition under which a supply profile over the operation horizon is adequate for a continuum of consumers subscribing to the considered DESs. We study the problem of allocating the available power to DES consumers to maximize social welfare. More importantly, we establish, in a forward market, the existence of an efficient competitive equilibrium. This fact indicates that self-interested DES consumers can attain the maximum social welfare, even in the absence of a social planner, thereby verifying the economic feasibility of the general duration-service-time DESs.

Remark. Prof. Pravin Varaiya has played a pivotal role in the birth and development of differentiated energy services, as evidenced by his unique contributions in [3, 4, 13, 20, 21]. Apart from smart grids, Prof. Varaiya has applied an economist's perspective to various domains, including transportation and networking [9]. The resulting influence has been demonstrated by a series of works, a few of which are listed here [6, 7, 8, 18, 19, 22, 23, 24, 26].

Notation. The symbol $\mathbf{1}$ denotes an all-one vector of a compatible dimension. For $n \in \mathbb{N}$, let $\underline{n}$ represent $\{1,2,\ldots,n\}$. The maximum of zero and a real number a is denoted by $[a]^+$. The function $\mathbb{1}(\mathfrak{A})$ maps a true assertion $\mathfrak{A}$ to one and a false assertion $\mathfrak{A}$ to zero.

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2.1. **Model formulation.** Discretize the operational period into n consecutive time slots, indexed from 1 to n. Then, partition these time slots into ν groups, where each group may contain a different number of time slots. Specifically, the κth group consists of the time slots indexed from $n_{\kappa-1}+1$ to n_κ, where $\kappa \in \underline{\nu}$ and $0 = n_0 < n_1 < \cdots < n_\nu = n$. Each DES is associated with a service time represented by $\boldsymbol{f} \in \{0,1\}^\nu$, where the service time includes the κth group of time slots if and only if $f_\kappa = 1$. Define $\bar{r}^{\boldsymbol{f}}$ as $[n_1 - n_0 \; n_2 - n_1 \; \cdots \quad n_\kappa - n_{\kappa-1}]\boldsymbol{f}$. Also, each DES with the service time $\boldsymbol{f}$ specifies a duration requirement $r \in \mathbb{N}$, satisfying $r \le \bar{r}^{\boldsymbol{f}}$. That is, a DES denoted by $(r, \boldsymbol{f})$ will deliver power during exactly r time slots included in the service time. The flexibility of this DES mainly lies in the difference between r and $\bar{r}^{\boldsymbol{f}}$. Consider m consumers and Consumer $i \in \underline{m}$ subscribes to a DES $(r_i, \boldsymbol{f}^i)$, for all $i \in \underline{m}$. Then, all duration requirements form the vector $\boldsymbol{r} \in \mathbb{N}^m$ and all service times of the m consumers are specified by the set $\mathscr{F} = \{\boldsymbol{f}^i \mid i \in \underline{m}\}$. The supply profile over the n-slot horizon is given by $\boldsymbol{c} \in \mathbb{N}^n$. Without loss of generality, we assume that in each time slot included in the service time of a DES, the supply delivers either no power or power at a constant rate to a load subscribing to the DES. One can refer to [21] for relating this abstract model to real data and see [10] for loads with varying maximum power delivery rates.

2.2. **Adequacy condition for discrete DES consumers revisited.** First of all, we wonder whether the supply $\boldsymbol{c}$ is adequate to satisfy the m consumers via the subscribed DESs. It is readily seen that the actual power allocation can be represented by an $m \times n$ $(0,1)$-matrix, where 1 means that the supply delivers power at a constant rate to the corresponding load at the associated time slot. A power delivery matrix $A = [a_{ij}] \in \{0,1\}^{m \times n}$ is said to be feasible if

$$c_j \ge \sum_{i \in \underline{m}} a_{ij}, \forall j \in \underline{n}; \tag{2.1}$$

$$r_i = \sum_{j \in \underline{n}} a_{ij}, \forall i \in \underline{m}; \tag{2.2}$$

$$a_{ij} = 0, \forall (i,j) \in \left\{ (i,j) \;\middle|\; \sum_{\kappa \in \underline{\nu}} f^i_\kappa \mathbb{1}(n_{\kappa-1} < j \le n_\kappa) = 0 \right\}, \tag{2.3}$$

where (2.1) means that the power delivered at each time slot should not exceed the available supply, the duration requirement of each load is satisfied by (2.2), and the formula in (2.3) ensures compliance with the service times of all subscribed DESs by imposing zero power delivery during time slots when the service is unavailable. As such, the start and completion of each service align with the predefined time windows specified for every DES. A necessary condition for the existence of a feasible power delivery matrix is that $\sum_{j\in\underline{n}} c_j \ge \sum_{i\in\underline{m}} r_i$. However, this condition is by no means sufficient due to the constraints on the constant power delivery rate and service times.

Let $\mathscr{A}([\boldsymbol{0}, \boldsymbol{c}], \boldsymbol{r}, \mathscr{F})$ denote the set of feasible power delivery matrices under the supply profile $\boldsymbol{r}$ and required DESs $(\boldsymbol{r}, \mathscr{F})$. In [10], a structure tensor is derived, whose nonnegativity is used to validate the non-emptiness of $\mathscr{A}([\boldsymbol{0}, \boldsymbol{c}], \boldsymbol{r}, \mathscr{F})$. For ease of notation, we assume that

$$c_{n_{\kappa-1}+1} \ge \cdots \ge c_{n_\kappa}, \text{ for all } \kappa \in \underline{\nu}.$$

Then, a structure tensor $W([\mathbf{0},\boldsymbol{c}],\boldsymbol{r},\mathscr{F})$ of dimension $(n_1-n_0+1)\times\cdots\times(n_\nu-n_{\nu-1}+1)$ is defined as follows:

$$W_{k_1k_2\cdots k_\nu}([\mathbf{0},\boldsymbol{c}],\boldsymbol{r},\mathscr{F})=\sum_{\kappa=1}^{\nu}\sum_{k=n_{\kappa-1}+k_\kappa+1}^{n_\kappa}c_j-\sum_{i=1}^{m}\left[r_i-[k_1\ k_2\ \cdots\ k_\nu]\boldsymbol{f}^i\right]^+, \tag{2.4}$$

where $0\leq k_j\leq n_j-n_{j-1}$, for all $j\in\underline{\nu}$.

In the following, we reproduce the structure-tensor condition in the DES context.

Theorem 2.1 ([10])**.** *The supply profile $\boldsymbol{c}$ is adequate to satisfy loads subscribing to DESs $\{(r_i,\boldsymbol{f}^i)\}$ if and only if $W([\mathbf{0},\boldsymbol{c}],\boldsymbol{r},\mathscr{F})\geq 0$, meaning each element of $W([\mathbf{0},\boldsymbol{c}],\boldsymbol{r},\mathscr{F})$ is nonnegative.*

Interested readers can refer to [10] for more details on the derivation of the structure tensor. Below is a proof sketch. First, the supply adequacy problem is transformed into a maximum *s*-*t* flow problem. According to the maximum-flow-min-cut theorem [5], the supply is adequate if and only if the capacity of each *s*-*t* cut exceeds the aggregated duration $\sum_{i\in\underline{m}}r_i$. However, there are an exponential number of *s*-*t* cuts. By leveraging the structure of $\mathscr{F}$, considerable inequalities are excluded, leading to the tensor condition.

Such a structure-tensor condition is particularly suitable for the supply/demand model, since the size of the tensor hardly grows as the number of rows (corresponding to demands) increases. It is clear that the tensor elements can be calculated in parallel. Nevertheless, recursive formulas could, in principle, be devised to compute the tensor elements efficiently, particularly when the number of processors is limited; however, such formulas are not presented in this paper. Given constrained computational resources, exploring parallel–sequential strategies for effective structure tensor computation remains a promising direction for future work. Moreover, we can utilize this tensor condition to find a feasible power delivery matrix if it exists.

3. A Forward Market Implementation

DESs result in a supply/demand balance, which cannot be assessed merely by comparing the total amounts of supply and demand, as traditionally done. Therefore, the economic feasibility of the new electricity services should be evaluated. To this end, consider DESs in a forward market, where all consumers subscribe to DESs before the operation horizon. We assume a continuum of consumers such that a single load is too tiny to impact the whole market. As mentioned in [20], this continuum assumption helps avoid bin-packing problems in load scheduling and eliminates assumptions about the concavity or uniformity of utility functions of different consumers. As initial efforts, we also disregard the uncertainties in supply and demand.

In the following, we list three key components in the forward market.

(1) Supply: The supplies over each operating cycle are given by $\boldsymbol{c}\in\mathbb{N}^n$.
(2) Services: In this market, there are only DESs in accordance with the slot grouping indices: $n_0,n_1,\ldots,n_\nu$. Let $\mathscr{S}$ denote the set of all types of DESs:

$$\mathscr{S}=\{(r\in\mathbb{N},\boldsymbol{f}\in\{0,1\}^n)\mid 0\leq r\leq\bar{r}^{\boldsymbol{f}}\}.$$

(3) Consumers: Consider a continuum of DES consumers indexed by the unit interval $[0,1]$. Each Consumer $x\in[0,1]$ demands $l(x)\in\mathbb{R}^+$ pieces of DES characterized by the duration requirement $r(x)$ and the service time $\boldsymbol{f}(x)$. Consequently, we can represent the

continuum of consumers by the following set and assume that $l(x), r(x)$ and $\boldsymbol{f}(x)$ are measurable on $[0,1]$:

$$\mathscr{R}^c = \{(l(x), r(x), \boldsymbol{f}(x)), x \in [0,1]\}.$$

3.1. **Adequacy condition for a continuum of DES consumers.** In this market of a continuum of consumers, a power delivery coordination can be represented by an infinite-dimensional matrix of n columns and a continuum of rows. Similarly to (2.1)-(2.3), a power delivery matrix $A = [a(x,j)] \in \{0,1\}^{[0,1]\times n}$ is said to be feasible if

$$\begin{aligned}
&c_j \geq \int_0^1 a(x,j) d_x, \forall j \in \underline{n};\\
&r(x) = \sum_{j\in\underline{n}} a(x,j), \forall x \in [0,1];\\
&a(x,j) = 0, \forall (x,j) \; \in \left\{ (x,j) \;\middle|\; \sum_{\kappa\in\underline{\nu}} f_\kappa(x) \mathbb{1}(n_{\kappa-1} < j \leq n_\kappa) = 0 \right\}.
\end{aligned}$$

The following theorem generalizes the structure-tensor condition for discrete DES consumers to that for a continuum of consumers regarding the existence of a feasible power delivery matrix.

Theorem 3.1. *The supply profile $\boldsymbol{c}$ is adequate for the continuum of DES consumers $\mathscr{R}^c$ if and only if $W^c([\mathbf{0},\boldsymbol{c}], \mathscr{R}^c) \geq 0$, where for all $k_\kappa = 0, 1, \ldots, n_\kappa - n_{\kappa-1}, \kappa \in \underline{\nu}$,*

$$W^c_{k_1 k_2 \cdots k_\nu}([\mathbf{0},\boldsymbol{c}], \mathscr{R}^c) = \sum_{\kappa=1}^{\nu} \sum_{k=n_{\kappa-1}+k_\kappa+1}^{n_\kappa} c_j - \int_0^1 l(x) \left[r(x) - [k_1 \; k_2 \; \cdots \; k_\nu] \boldsymbol{f}(x) \right]^+ dx. \tag{3.1}$$

It is clear that Formula (3.1) is consistent with Formula (2.4), by replacing the summation for discrete DES consumers with the integration for the continuum of DES consumers. Consequently, the correctness of Theorem 3.1 naturally follows from Theorem 2.1 and the definition of a continuum of loads. This fact further highlights the effectiveness of the structure-tensor condition, since the derivation of the tensor condition differentiates the roles of supply and demand, with the tensor size remaining independent of the number of consumers subscribing to each type of DES.

3.2. **Social welfare maximization.** Before proceeding, for all $x \in [0,1]$, let $U(x, l(x), r(x), \boldsymbol{f}(x))$ (with $U(x, 0, r(x), \boldsymbol{f}(x)) = 0$) denote the uniform utility function (non-negatife, bounded, and continuous) and consider the following social welfare maximization problem.

$$\begin{aligned}
\max_{l(x), r(x), \boldsymbol{f}(x)} \; & \int_0^1 U(x, l(x), r(x), \boldsymbol{f}(x)) dx \\
\text{s.t.} \quad & W^c([\mathbf{0},\boldsymbol{c}], \mathscr{R}^c) \geq 0; l(x) \geq 0, (r(x), \boldsymbol{f}(x)) \in \mathscr{S}, \forall x \in [0,1].
\end{aligned} \tag{3.2}$$

Similarly to [20] for duration-differentiated energy services and [13] for multiple-arrival multiple-deadline differentiated energy services, the next goal is to show the existence of an optimum for an arbitrary measurable utility function, following the convention in the study of an economy with a continuum of traders [2].

To achieve this goal, define a tensor-valued function $Z(x) = [Z_{k_1k_2\cdots k_\nu}(x)]$ with its initialization as $Z(0) = O$ and its derivative with respect to x as $\dot{Z}(x) = [\omega_{k_1k_2\cdots k_\nu}(x)]$, where

$$\omega_{k_1k_2\cdots k_\nu}(x) = \begin{cases} l(x)\,[r(x) - [k_1\ k_2\ \cdots\ k_\nu]\boldsymbol{f}(x)]^+, \ \forall\, 0 \le k_\kappa \le n_\kappa - n_{\kappa-1}, \kappa \in \underline{\nu}; \\ U(x, l(x), r(x), \boldsymbol{f}(x)), \ \forall\, k_1 = -1, 0 \le k_\kappa \le n_\kappa - n_{\kappa-1}, 2 \le \kappa \le \nu. \end{cases}$$

It follows that a set-valued function $x \mapsto \Omega(x)$ is obtained, where

$$\Omega(x) = \{\omega(x) | l(x) \ge 0, (r(x), \boldsymbol{f}(x)) \in \mathscr{S}\},$$

and $\dot{Z}(x) \in \Omega(x), \forall x \in [0,1]$. By the integration of set-valued functions [2], it leads to the integral of the set-valued correspondence

$$\mathscr{G} = \int_0^1 \Omega(x)dx = \{Z(1) \mid Z(1) \text{ is reached by a feasible power delivery for } x \mapsto (l(x), \boldsymbol{f}(x)) \in \mathscr{S}\}.$$

It follows from the main theorems in [2] that $\mathscr{G}$ is a convex and closed set. Then, restate the welfare maximization problem (3.2) in terms of $\mathscr{G}$ as

$$\begin{aligned} &\max_{Z(1)} Z_{(-1)0\cdots0}(1) \\ &\text{s.t.} \quad Z(1) \in \mathscr{G}; Z_{k_1k_2\cdots k_\nu}(1) \le \sum_{\kappa=1}^{\nu} \sum_{k=n_{\kappa-1}+k_\kappa+1}^{n_\kappa} c_j, \forall\, 0 \le k_\kappa \le n_\kappa - n_{\kappa-1}, \kappa \in \underline{\nu}. \end{aligned} \tag{3.3}$$

Proposition 3.2. *The optimum of the welfare maximization problem* (3.2) *exists.*

Proof. It follows from the properties of $\mathscr{G}$ and the linear constraints regarding the summation of partial supplies that the feasible set of Problem (3.3) is convex and compact. Thus, the theorem follows from the fact that the optimal solution $Z^*(1)$ exists. □

3.3. **Competitive equilibrium.** A focus of this note is to study the competitive equilibrium in a forward market with the considered DESs. Assume that all market participants, including the supplier and consumers, are rational and self-interested price-takers. Denote the price for a portion of the DES $(r, \boldsymbol{f})$ by $\pi_r^{\boldsymbol{f}}$. It is evident that $\pi_0^{\boldsymbol{f}} = 0$ for all $\boldsymbol{f} \in \{0,1\}^\nu$. In a forward market with a continuum of consumers, it is assumed that an individual transaction does not influence the prices. Moreover, a competitive equilibrium should satisfy the following three conditions.

(1) Each consumer maximizes its welfare. That is, each consumer $x \in [0,1]$ solves the following optimization problem:

$$\max_{l(x) \ge 0, (r(x), \boldsymbol{f}(x)) \in \mathscr{S}} U(x, l(x), r(x), \boldsymbol{f}(x)) - l(x)\pi_{r(x)}^{\boldsymbol{f}(x)} \tag{3.4}$$

(2) The supplier maximizes its revenue. Let $q_r^{\boldsymbol{f}}$ denote the quantity of the DES $(r, \boldsymbol{f}) \in \mathscr{S}$ produced by the supplier to maximize its revenue. Then, the revenue maximization

problem is formulated as

$$\begin{aligned}
&\max_{q_r^{\boldsymbol f},\delta_r^{\boldsymbol f}} \sum_{\boldsymbol f\in\{0,1\}^\nu}\sum_{r=1}^{\bar r^{\boldsymbol f}} q_r^{\boldsymbol f}\pi_r^{\boldsymbol f}\\
&\text{s.t. } \delta_j^{\boldsymbol f}=\sum_{r=1}^{\bar r^{\boldsymbol f}} q_r^{\boldsymbol f}\\
&\qquad \sum_{\kappa=1}^{\nu}\sum_{j=n_{\kappa-1}+k_\kappa+1}^{n_\kappa} c_j-\sum_{\boldsymbol f\in\{0,1\}^\nu}\sum_{j=1}^{\bar r^{\boldsymbol f}}\delta_j^{\boldsymbol f}\ge 0, 0\le k_\kappa\le n_\kappa-n_{\kappa-1}, \kappa\in\underline{\nu}.
\end{aligned}\tag{3.5}$$

(3) The market is clear. That is,

$$q_r^{\boldsymbol f}=\int_0^1 l(x)\mathbb{1}\left(r(x)=r \text{ and } \boldsymbol f(x)=\boldsymbol f\right)dx, \text{ for each } (r,\boldsymbol f)\in\mathscr{S} \tag{3.6}$$

Clearly, a competitive equilibrium is attained in a distributed manner, with each market participant optimizing its own benefit while supply matches demand. A competitive equilibrium is said to be **efficient** if the resource allocation at the equilibrium maximizes social welfare.

Theorem 3.3. *There exists an efficient competitive equilibrium in a forward market for DESs.*

Proof. We first show that there exists a menu of prices $\{\pi_r^{\boldsymbol f}\mid(r,\boldsymbol f)\in\mathscr{S}\}$ such that Problem (3.4) and Problem (3.5) admit optimal solutions satisfying (3.6). To this end, denote the optimal solutions to Problem (3.2) by $\{(\hat l(x),\hat r(x),\hat a(x),\hat d(x))\mid x\in[0,1]\}$. It is seen that this resource allocation $x\in[0,1]\mapsto(\hat l(x),\hat r(x),\hat a(x),\hat d(x))$ forms an optimal solution $\hat Z(1)$ to the restated problem (3.3). Considering Problem (3.3), there exist Lagrange multipliers:

$$\{\alpha_{k_1k_2\cdots k_\nu}\ge 0\mid 0\le k_\kappa\le n_\kappa-n_{\kappa-1},\kappa\in\underline{\nu}\}$$

such that $\hat Z(1)$ optimally solve the following problem

$$\max_{Z(1)\in\mathscr{G}} Z_{-1\,0\,\cdots\,0}(1)-\sum_{k_1,k_2,\cdots,k_\nu}\alpha_{k_1k_2\cdots k_\nu}Z_{k_1k_2\cdots k_\nu}(1). \tag{3.7}$$

Moreover, the Lagrange multipliers and optimal solution respects the complementary slackness; that is, for all $0\le k_\kappa\le n_\kappa-n_{\kappa-1}, \kappa\in\underline{\nu}$, it holds that

$$\alpha_{k_1k_2\cdots k_\nu}\left[X^*_{k_1k_2\cdots k_\nu}(1)-\left(\textstyle\sum_{j>k_1}^{k_1}c(j)+\cdots+\sum_{j>n_{\nu-1}+k_\nu}^{k_\nu}c(j)\right)\right]=0$$

Define

$$\hat\pi_{r(x)}^{\boldsymbol f(x)}=\sum_{k_1,k_2,\cdots,k_\nu}\alpha_{k_1k_2\cdots k_\nu}\left[r(x)-[k_1\ k_2\ \cdots\ k_\nu]\boldsymbol f(x)\right]^+, \tag{3.8}$$

and then the term to be maximized in (3.7) can be restated as

$$\int_0^1 U(x,l(x),r(x),\boldsymbol f(x))-l(x)\hat\pi_{r(x)}^{\boldsymbol f(x)}dx.$$

According to the complementary slackness, the adequacy condition in Theorem 3.1, and similar techniques in [13], we can show that each consumer maximizes its utility, the supplier maximizes its revenue, and the market is clear under the prices defined by (3.8) and the resource allocation $\{(\hat l(x),\hat r(x),\hat a(x),\hat d(x))\mid x\in[0,1]\}$. It follows that a competitive equilibrium is attained at such menu of prices (3.8) and resource allocation $\{(\hat l(x),\hat r(x),\hat a(x),\hat d(x))\mid x\in[0,1]\}$.

Moreover, since $\{(\hat{l}(x), \hat{r}(x), \hat{a}(x), \hat{d}(x)) \mid x \in [0,1]\}$ is an optimal solution to the social welfare maximization problem (3.2), we conclude that this competitive equilibrium is efficient. □

The existence and performance of a competitive equilibrium are often used to measure the long-term market efficiency [1]. Particularly, an efficient competitive equilibrium indicates that self-interested DES consumers attain the maximum social welfare, even in the absence of a social planner. In other words, resources will be allocated efficiently in the long run within a competitive market. Therefore, Theorem 3.3 can validate the economic feasibility of the involved services to a certain degree.

4. Conclusion and Future Work

In this paper we examined energy services differentiated by duration requirements and service times, following the research line on duration-differentiated energy services [20], duration-deadline jointly differentiated energy services [4], and multiple-arrival multiple-deadline differentiated energy services [13]. We derived a structural tensor condition to assess the adequacy of a supply for a continuum of loads consuming the considered DESs. Furthermore, we explored the market implementation and verified the economic feasibility of these DESs by proving the existence of an efficient competitive equilibrium.

Further research inspired by this work includes three directions. First, while the theoretical analysis of DESs has been established, their practical application remains a challenging yet meaningful topic. Second, we expect to elevate the study of the structure-tensor condition for more complicated matrix completion such as $(-1,0,1)$-matrix [11] and advanced theories like partial order programming [17]. Finally, it is of interest to apply the service model to other fields. Notably, Prof. Pravin Varaiya also consistently paid attention to the demand-side contributions to power, transportation, and networking systems [3, 6, 7, 8, 13, 18, 19, 20, 21, 22, 23, 24, 26].

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