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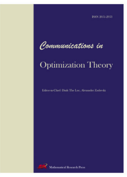

DYNAMIC WELFARE ANALYSIS OF INCOME DISTRIBUTIONS: THE TRADE-OFF BETWEEN EQUITY AND LONG-RUN EFFICIENCY

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This paper is a tribute to a scholar who has made an indelible mark on my personal and professional growth. It is dedicated to the memory of my teacher and mentor, Professor Charalambos Aliprantis, on the occasion of his 80th birthday

Abstract. This paper exploits the theory of stochastic orders to formalize preferences over sequences of income distributions that embody suitable principles of equity and efficiency. Intragenerational and intergenerational equity are combined into a new concept of equity that extends the Hammond's equity principle (for infinite utility streams) to the realm of stochastic processes. The main result of the paper can be interpreted as follows: short-sighted, piecemeal economic policies that enhance equity may come at the expense of the income of generations in the far-off future.The proof of the instrumental lemma could be of interest in its own right because it uncovers the interplay between moment-generating functions and stochastic orders.

Keywords. Efficiency; Income distribution; Moment-generating function; Stochastic orders; Sequences of random variables; The equity principle.

2020 Mathematics Subject Classification. 60E15, 62P20, 91B15.

1. INTRODUCTION

It is often argued that policy-makers are short-sighted and use piecemeal policies because they prioritize policies that win public support (e.g., due to short election cycles) and enact fragmented solutions to complex challenges. My objective is to investigate the implications of short-sightedness and piecemeal policies for welfare analysis of time-sequences of income distributions and for their evolution over time. Specifically, I am interested in the following scenario: a status-quo trajectory of income distributions is exogenously given. There is a short-sighted policy-maker (i.e., a policy-maker with a two-period planning horizon) that must choose between two options: doing nothing, i.e., maintaining the status-quo sequence of income distributions, or changing it to make it 'more equitable'. If the policy-maker opts for changing the status-quo sequence, it takes action with piecemeal policies. Some questions naturally arise: what does it mean to change from the status-quo sequence to a new, more equitable sequence

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of income distributions by implementing piecemeal policies, and what does a more equitable sequence look like? If there exists a dynamic path (for the income distributions) more equitable than the status-quo, and the policy-maker puts the economy on this new path, how does the new sequence compare to the status-quo, efficiency-wise? Could 'increasing' equity entail giving up efficiency, presumably in the 'long-run'?

In a dynamic, multi-person environment, the definition of equity used for economic analysis should combine notions of intragenerational and intergenerational equity. Therefore, I will take into consideration efficiency (which refers to the maximization of income at any point in time), intragenerational equity (which refers to the variability of income across the population of a given generation), and intergenerational equity (i.e., 'fairness' across different generations, or aversion to unequal 'welfare' levels of different generations). I develop a mathematical framework for modeling the scenario and addressing the questions set out above. Toward this end, I take advantage of the following known fact: there is a one-to-one correspondence between cumulative distribution functions defined on the real numbers and real-valued random variables on $(0,1)$ endowed with the Lebesgue measure. Therefore, provided that the income distributions under consideration have a closed-form cumulative distribution function, I can treat them as random variables defined on the same probability space[1] and exploit the theory of stochastic orders for the analysis of preferences over sequences of random variables. Incidentally, working with random variables defined on the same probability space makes the notion of more efficient income distribution particularly compelling, and simplifies the task of finding a random variable Z which satisfies (7) in Section 2.

Specifically, I assume that the status-quo sequence of random variables is 'increasing' and 'bounded from above' (with respect to the usual stochastic order) and converges in distribution. I formalize and justify this assumption in Section 4 (see Assumption 1 and Remark 3). I introduce a binary (preference) relation over sequences of random variables (the policy-maker is endowed with) and I assume that it satisfies the equity principle, which is a new principle I come up with relying on the so-called increasing concave order (see Definition 1 in Section 3). The equity principle is inspired by the Hammond's equity principle for infinite utility streams; it captures aversion to intergenerational inequalities and also incorporates the intragenerational equity principle previously mentioned. At the same time, I define a notion of 'more efficient' income distribution based on the so-called usual stochastic order, and I introduce the concept of equity-efficiency trade-off in the long run which will be used in Theorem 1 to adress the issue of efficiency in the long-run versus equity (see also Remarks 4 and 5). This way, one can say that equity goes up if the economy switches to a new sequence of random variables which is (weakly) preferred to the one at hand according to the aforementioned preferences, and one can also explore whether efficiency 'goes down' in the pursuit of equity.

As a matter of fact, the thrust of Theorem 1 (the main result) can be summarized as follows: suppose that a policy-maker enhances equity of income distributions over time using a piecemeal approach, i.e., the transition from the status-quo sequence of income distributions to a new sequence aimed at by the policy maker with a two-period planning horizon (i.e., a short-sighted policy maker) takes place through infinitely-many sequential steps. There exists a new sequence, which could be targeted by the policy-maker, such that each such step results

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[1] Note that the evolution over time of the distribution of income had been already modeled as a stochastic process (see, e.g., [2]).

in a sequence of random variables which is more equitable than the one from the previous step. Furthermore, if the new sequence converges in distribution the status-quo sequence is asymptotically 'more efficient' than the new sequence, i.e., the limit random variable of the status-quo sequence is 'greater than' (in the usual stochastic order sense) the random variable which is the limit of the new sequence. This shows that there can exist a trade-off between equity and long-run efficiency even if the two sequences are not comparable according to the monotonicity principle (see Remarks 4 and 5, in Section 4, for a more elaborate interpretation of Theorem 1). Incidentally, the proof of Lemma 1, which is instrumental for Theorem 1, hinges on a result pertaining to the theory of moment-generating functions. This tool comes naturally into play because the definition of moment-generating function and the definition of the stochastic orders used in this work fit together very well (see also Proposition 1 in Section 5).

In a static setting, there is a vast literature on the use of stochastic orders for the analysis of income distribution problems, inequality measures, and decision-making under risk (see, e.g., [7, 9]). In a dynamic environment, axiomatic analyses of social welfare orders on stochastic processes and their representability by welfare functions have already been examined (see, e.g., [5]). Note that in [5] only the usual stochastic order is featured, which is employed to define the monotonicity principle for stochastic processes (see Remark 2, in Section 3, for the definition of the monotonicity principle). The novelty of my contribution is that I study sequences of income distributions and I work with a richer set of stochastic orders in a full-fledged manner (i.e., I exploit the mathematical characterization of various stochastic orders and some results on how they are related to each other). Additionally, the method I use to investigate the existence of an equity-efficiency tradeoff is different from the standard approach in the existing literature which consists in proving that social welfare orders satisfying suitable efficiency and equity properties turn out to be non-constructive objects whenever they exist (see, e.g., [4, 6]). The reason I deviate from this traditional methodology is twofold: firstly, as I argue in Remark 2, in Section 3, if one is willing to give up the requirement that preferences be complete, Rawlsian preferences are both 'efficient' and 'equitable'. Therefore, the aforementioned standard approach would not work in my model; secondly, I am mostly interested in showing that short-sighted, piecemeal economic policies that boost equity may come at the expense of the income of future generations.

The paper is structured as follows: in Section 2, I gather some results, regarding stochastic orders, that will be utilized throughout the paper. In Section 3, I introduce criteria for equity and efficiency, I explain the concept of trade-off between efficiency and equity in the long run, and define formally the equity principle. I also exhibit a transitive preference relation that respects the equity principle (see Remark 2). In Section 4, I state the assumption my model rests on and discuss the motivation behind it, and then I prove the main result. To streamline its proof, I accomplish my goal in two steps (i.e., Lemma 1, and Theorem 1). Remark 4 offers an economic interpretation of Theorem 1, and I also construct transitive pereferences (satisfying the equity principle) which are used to clarify the scope of Theorem 1 (see Remark 5). In Section 5, I present two examples that illustrate Theorem 1. Section 6 concludes the paper with a few remarks on open questions and avenues for future research.

2. BACKGROUND

In this section, I will overview several definitions and results about stochastic orders that will be used in Sections 3, 4 and 5 of the paper. They draw extensively on [10], to which I refer the reader for a thorough account and the details that will be omitted here. I will also state a useful result about the moment generating function of a random variable. Throughout the paper E will denote the expected value.

Let X and Y be two real-valued random variables (not necessarily defined on the same probability space). The so-called usual stochastic order (for its definition, see [10, Section 1.A.1]) will be denoted by $\geq_{st}$. So, $Y \geq_{st} X$ means that X is smaller than Y in the usual stochastic order. One can prove (see [10, expression 1.A.7]) that $Y \geq_{st} X$ if and only if

$$E[\phi(X)] \leq E[\phi(Y)] \tag{1}$$

holds for all nondecreasing functions $\phi : \mathbb{R} \to \mathbb{R}$ for which the expectations exist. Next, I will state somewhat loosely [10, Theorem 1.A.1] which provides an insightful characterization of the usual stochastic order. I will rely on it to formalize the concept of more efficient income distribution in Section 3.

X and Y satisfy $Y \geq_{st} X$ if and only if there exist two random variables $\hat{X}$ and $\hat{Y}$, defined on the same probability space, such that $\hat{X}$ and $\hat{Y}$ are equal in law to X and Y, respectively, and the probability that $\hat{Y} \geq \hat{X}$ is equal to 1.

The following closure property of the usual stochastic order will be used in the proof of Lemma 1, in Section 4. For the proof, see [10, Theorem 1.A.3, part (c)]:

Let $\{X_n\}_{n=1}^{\infty}$ and $\{Y_n\}_{n=1}^{\infty}$ be two sequences of random variables such that $X_n \xrightarrow{d} X$ and $Y_n \xrightarrow{d} Y$ as $n \to \infty$, where '$\xrightarrow{d}$' denotes convergence in distribution. If $X_n \leq_{st} Y_n$, $n = 1, 2,...,$ then $Y \geq_{st} X$.

Let X and Y be two real-valued random variables (not necessarily defined on the same probability space) such that

$$E[\phi(X)] \leq E[\phi(Y)] \text{ for all convex } \phi : \mathbb{R} \to \mathbb{R}, \tag{2}$$

provided the expectations exist. Then, X is said to be smaller than Y in the convex order (denoted as $Y \geq_{cx} X$). One can also define a concave order by requiring (2) to hold for all concave functions $\phi : \mathbb{R} \to \mathbb{R}$ (denoted as $Y \geq_{cv} X$). It is straightforward to prove that

$$Y \geq_{cv} X \text{ if and only if } X \geq_{cx} Y. \tag{3}$$

Note that (3) easily implies (see [10, (3.A.2)])

$$Y \geq_{cv} X \implies E[Y] = E[X], \tag{4}$$

provided the expectations exist. Moreover, (3) implies (see [10, (3.A.4)])

$$Y \geq_{cv} X \implies \text{Variance of } Y \leq \text{Variance of } X. \tag{5}$$

(4) and (5) will be invoked in the next section to rationalize the definition of intragenerational equity. Let X and Y be two real-valued random variables (not necessarily defined on the same probability space) such that

$$E[\phi(X)] \leq E[\phi(Y)] \text{ for all nondecreasing and concave [convex] } \phi : \mathbb{R} \to \mathbb{R}, \tag{6}$$

provided the expectations exist. Then, X is said to be smaller than Y in the increasing concave [convex] order (denoted by $Y \geq_{icv} X$ $[Y \geq_{icx} X]$). Loosely speaking, if $Y \geq_{icx} X$ then X is both 'smaller' and 'less variable' than Y. Similarly, if $Y \geq_{icv} X$ then X is both 'smaller' and 'more variable' than Y. It is very easy to see that $Y \geq_{st} X$ implies both $Y \geq_{icv} X$ and $Y \geq_{icx} X$. Similarly, $Y \geq_{cv} X$ implies $Y \geq_{icv} X$. In what follows, I will put on record a characterization result for the increasing concave order which, in Section 4, will play a key role in the proof of Theorem 1. For the proof, see [10, Theorem 4.A.6, part (c)]:

Two random variables X and Y satisfy $X \leq_{icv} Y$ if and only if there exists a random variable Z such that

$$X \leq_{cv} Z \leq_{st} Y. \tag{7}$$

The following convergence theorem is originally due to [3] and will be employed in the proofs of Lemma 1 and Proposition 1:

Let $\{X_n\}_{n=1}^{\infty}$ be a sequence of random variables whose moment-generating functions $M_{X_n} : \mathbb{R} \to \mathbb{R}$ are well-defined on some common neighbourhood $(-\delta, \delta)$ of zero. Suppose that $\{M_{X_n}\}_{n=1}^{\infty}$ converges pointwise to some function $M : \mathbb{R} \to \mathbb{R}$ on $(-\delta, \delta)$. Then, there exists a unique random variable X such that $\{X_n\}_{n=1}^{\infty}$ converges to X in distribution, and M is the moment-generating function of X on $(-\delta, \delta)$.

In the remainder of the paper, it will be assumed that the relevant moments of all random variables at hand exist.

3. THE INFINITE-HORIZON ECONOMY: EFFICIENCY AND EQUITY

There are countably-many generations, denoted by $n \in \mathbb{N}$. Each generation n is composed of (possibly) infinitely-many individuals, with the income distribution of each generation being described by a random variable (as I have explained in the introduction).

I will begin by formalizing the concepts of 'more efficient' income distribution and 'more intragenerationally equitable' income distribution using stochastic orders. I will then turn to a binary relation over sequences of random variables (i.e., income distributions) which will be interpreted as a preference relation. For this preference relation I will define the equity principle (see Definition 1), which is a principle of intergenerational equity that reflects aversion to intergenerational inequalities and incorporates the principle of intragenerational equity. Finally, I will rely on the definition of 'more efficient' income distribution to explain the concept of trade-off between efficiency and equity in the long run.

Efficiency: Let X and Y be two real-valued random variables defined on the same probability space. Suppose that $Y \geq_{st} X$. Then, an immediate consequence of (1) is that $E[Y] \geq E[X]$. Furthermore, the characterization of the usual stochastic order reported in the previous section suggests, roughly, that almost everyone has more income in a generation with income distribution Y than in a generation with income distribution X. Therefore, in moving from X to Y, income, overall, increases. In conclusion, it seems natural to assume that if the income distribution changes from X to Y efficiency goes up. This definition will be used to conceptualize long-run efficiency and examine the potential trade-off with equity (see below at the end of this section, and also Theorem 1 and Remarks 4 and 5).

Intragenerational equity: Let X and Y be two real-valued random variables. Suppose that $Y \geq_{cv} X$. Then, by (4) and (5) above one can assume that (income distribution) Y is more equitable than (income distribution) X from the point of view of intragenerational equity. To reinforce the rationale for this assumption, note that in the case in which the supports of X and Y are intervals, (3) used in conjunction with [10, $(3.A.12)$] yields the following result: if $Y \geq_{cv} X$, the support of Y is a subset of the support of X.

The equity principle: Suppose that $Y \geq_{icv} X$. Using (6) it is very easy to see that $E[Y] \geq E[X]$. Moreover, it follows from (7) that there exists a random variable Z such that $X \leq_{cv} Z \leq_{st} Y$. Thus, (income distribution) Y is more efficient than (income distribution) Z, and, in turn, (income distribution) Z is more intragenerationally equitable than (income distribution) X. Therefore, it seems reasonable to assume that if $Y \geq_{icv} X$, (income distribution) Y is 'better' than (income distribution) X. This suggests that if intragenerational equity is to be taken into accout and X_i and X_j are the income distributions of two different generations, i and j, with $X_j \geq_{icv} X_i$, then generation j is better off than generation i. Based on this insight I will formulate an equity principle which extends the Hammond's equity principle to sequences of random variables. In terms of notation, keep in mind that '$=_{st}$', which appears in the following definition, denotes equality in law.

Definition 1. (**The equity principle**). Let $\succeq$ be a reflexive[2] binary (preference) relation over sequences of random variables. $\succeq$ is said to satisfy the equity principle if, whenever $\{X_n\}_{n=1}^{\infty}$ is a sequence of random variables such that $X_j \geq_{icv} X_i$ for some i and j, with $i \neq j$, and $\{Y_n\}_{n=1}^{\infty}$ is another sequence of random variables satisfying $Y_n =_{st} X_n$ for all $n \neq i$ and j and $X_i \leq_{icv} Y_i \leq_{icv} Y_j \leq_{icv} X_j$, $\{Y_n\}_{n=1}^{\infty} \succeq \{X_n\}_{n=1}^{\infty}$ holds.

Remark 1. To see that Definition 1 is well-posed, observe that $Y \geq_{st} X$ implies $Y \geq_{icv} X$, and $Y \geq_{cv} X$ implies $Y \geq_{icv} X$ (see Section 2). Therefore, it follows from (7) that if $X_j \geq_{icv} X_i$ there exists a random variable Y_i such that $X_i \leq_{icv} Y_i \leq_{icv} X_j$, which implies $X_i \leq_{icv} Y_i \leq_{icv} Y_j \leq_{icv} X_j$, where $Y_j = Y_i$.

Remark 2. I will construct transitive preferences that satisfy the equity principle. They can be interpreted as Rawlsian preferences: restrict attention to the set of sequences of random variables $\mathbf{x} = \{X_n\}_{n=1}^{\infty}$ with the following properties: $E[X_n] \in \mathbb{R}$ for all n, and there exists $C_{\mathbf{x}} \in \mathbb{R}$ such that $C_{\mathbf{x}} \leq E[X_n]$ for all n. Call such a set S. Observe that the status-quo sequence $\{X_n\}_{n=1}^{\infty}$ considered next in this paper belongs to S (refer to Assumption 1, in Section 4). Similarly, it turns out that also the sequence $\{Y_n\}_{n=1}^{\infty}$, the existence of which is proven in Theorem 1, is an element of S. Consider the preference relation $\succeq$ over S defined by: for all $\{Z_n\}_{n=1}^{\infty}$ and $\{Q_n\}_{n=1}^{\infty} \in S$, $\{Z_n\}_{n=1}^{\infty} \succeq \{Q_n\}_{n=1}^{\infty}$ if and only if $\inf(\{E[Z_n]\}_{n=1}^{\infty}) \geq \inf(\{E[Q_n]\}_{n=1}^{\infty})$. It is not difficult to verify that this preference relation satisfies the equity principle and, of course, is transitive. Interestingly, let's say that if $A_n \geq_{st} B_n$ for all $n \geq 1$, $\{A_n\}_{n=1}^{\infty}$ is weakly-preferred to $\{B_n\}_{n=1}^{\infty}$ according to the monotonicity principle (which is an efficiency principle). Then, it is very easy to show that the above Rawlsian preferences respect also the monotonicity principle.

Equity-efficiency trade-off in the long run: Suppose that the two sequences of random variables $\{X_n\}_{n=1}^{\infty}$ and $\{Y_n\}_{n=1}^{\infty}$ represent, respectively, the status-quo time-sequence of income distributions (i.e., the evolution of income distributions if the policy-maker does not intervene) and the time-sequence of income distributions that results from policy-making that fosters equity.

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[2] It is easy to verify that reflexivity is a necessary condition for the equity principle. This is why I require preferences to be reflexive.

Suppose that $\{X_n\}_{n=1}^{\infty}$ and $\{Y_n\}_{n=1}^{\infty}$ converge in some mathematical way to the random variables $\bar{X}$ and $\bar{Y}$, respectively. If $\bar{X} \geq_{st} \bar{Y}$, then in the long run there is a trade-off between efficiency and equity.

4. THE MAIN RESULT

In this section, I will state and discuss the assumption of my model. I will then turn to the main result of this paper, i.e., Theorem 1. The proof of it will be accomplished in two steps: Lemma 1 will be established first to streamline the proof of Theorem 1. I will give a proof of Theorem 1 by induction which is susceptible of a simple economic interpretation: existence of an 'inverse' relationship between equity and efficiency in the long-run (see Remark 4 below). Finally, I will construct specific preferences which I will use to clarify the scope of Theorem 1 (see Remark 5 below).

Assumption 1. The status-quo sequence of income distributions is represented by a sequence of random variables $\{X_n\}_{n=1}^{\infty}$ which satisfies the following properties: (i) $X_{n+1} \geq_{st} X_n$ for all $n \geq 1$, and there exists a random variable $\hat{X}$ such that $\hat{X} \geq_{st} X_n$ for all $n \geq 1$. (ii) $\{X_n\}_{n=1}^{\infty}$ converges in distribution to a random variable $\bar{X}$.

Remark 3. The logic behind Assumption 1 can be explained as follows: an easy consequence of (1), applied to the sequence of Assumption 1, is that $E[X_n] \leq E[X_{n+1}]$. Moreover, recall from Section 2 that $X_{n+1} \geq_{st} X_n$ implies both $X_{n+1} \geq_{icv} X_n$ and $X_{n+1} \geq_{icx} X_n$. The interpretation of these 'inequalities' is that the real gross domestic product per capita grows over time and, as income per capita increases, the income distribution can get more or less variable, with no clear-cut relationship between inequality and income per capita. Therefore, on the one hand Assumption 1 reflects the increase of income per capita over time stipulated by standard models of long-run growth. On the other hand, it is a way to take an agnostic stand about the relationship between income inequality and economic growth. Arguably, it is appropriate to take an agnostic stand in view of the so-called Kuznets curve and given the conflicting evidence on such relationship (to the best of my knowledge.). Note, however, that standard models of long-run economic growth (see, e.g., [1, 8, 11, 12]) also predict that at the steady-state the growth of income per capita is determined by the rate of technological progress. This feature of long-run growth may be at variance with the assumption that the sequence of random variables (modeling the evolution of income distributions) is bounded above. So, it would be worth exploring if this assumption can be dispensed with, as part of a future research project.

Lemma 1. *Let* $\{X_n\}_{n=1}^{\infty}$ *be a sequence of random variables satisfying Assumption 1. Let* $\{Y_n\}_{n=1}^{\infty}$*be another sequence of random variables satisfying the following properties*: $X_1 \leq_{cv} Y_1$, $Y_n \leq_{cv} Y_{n+1}$ *for all* $n \geq 1$, *and* $Y_n \leq_{st} X_n$ *for all* $n \geq 2$. *Then, if either the moment-generating function of* X_1 *exists on a neighborhood of zero* $(-\delta, \delta)$, *or* $\{Y_n\}_{n=1}^{\infty}$*converges in distribution,* $\{Y_n\}_{n=1}^{\infty}$*converges in distribution to* $\bar{Y}$ *and* $\bar{X} \geq_{st} \bar{Y}$.

Proof. First, suppose that the moment-generating function of X_1, $M_{X_1} : \mathbb{R} \to \mathbb{R}$, is well-defined on $(-\delta, \delta)$. I claim that for all $n \geq 1$ the moment-generating function of Y_n, $M_{Y_n} : \mathbb{R} \to \mathbb{R}$, exists on the open neighborhood of zero $(-\delta, \delta)$. To see this, use the assumed properties of $\{Y_n\}_{n=1}^{\infty}$ together with transitivity of the the concave order (see (2)) to get

$$X_1 \leq_{cv} Y_n \text{ for all } n \geq 1. \tag{8}$$

(3) and (8) immediately imply

$$X_1 \geq_{cx} Y_n \text{ for all } n \geq 1. \tag{9}$$

Define $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = e^{tx}$, and note that for all $t \in (-\delta, \delta)$ f is convex (as a function of x). Also, by assumption the moment-generating function $M_{X_1}(t)$ is well-defined for all $t \in (-\delta, \delta)$. Therefore, it follows from (2) and (9) that

$$M_{X_1}(t) := E\left[e^{tX_1}\right] \geq E\left[e^{tY_n}\right] \text{ for all } n \geq 1, \text{ and for all } t \in (-\delta, \delta). \tag{10}$$

With the definition of moment-generating function in mind, (10) establishes the desired claim. Next, note that by assumption $Y_n \leq_{cv} Y_{n+1}$ for all $n \geq 1$. Therefore, (3) yields $Y_n \geq_{cx} Y_{n+1}$ for all $n \geq 1$. Since the existence of the moment-generating functions of the random variables Y_n has been already proven, the previous 'inequality', used in conjunction with (2) and the definition of moment-generating function, yields

$$M_{Y_n}(t) \geq M_{Y_{n+1}}(t) \text{ for all } n \geq 1 \text{ and for all } t \in (-\delta, \delta). \tag{11}$$

Recall that $X_1 \leq_{cv} Y_n$ for all $n \geq 1$. Thus, (4) implies

$$E[Y_n] = E[X_1] \text{ for all } n \geq 1. \tag{12}$$

Moreover, Jensen's inequality provides the following lower bound for a random variable X with a well-defined moment-generating function: $M_X(t) \geq e^{tE[X]}$ for all t. Therefore, (12) implies

$$M_{Y_n}(t) \geq e^{tE[X_1]} \text{ for all } n \geq 1, \text{ and for all } t \in (-\delta, \delta), \tag{13}$$

where $e^{tE[X_1]} \in \mathbb{R}$ because $E[X_1]$ clearly exists. (11) and (13) imply that $\{M_{Y_n}\}_{n=1}^{\infty}$ converges pointwise on $(-\delta, \delta)$. Thus, by Curtiss's convergence theorem (see the end of Section 2) $\{Y_n\}_{n=1}^{\infty}$ converges in distribution to a unique random variable $\bar{Y}$. Now, by Assumption 1 $\{X_n\}_{n=1}^{\infty}$ converges in distribution to $\bar{X}$, and it has just been proven that $\{Y_n\}_{n=1}^{\infty}$ converges in distribution to $\bar{Y}$. Therefore, also the two subsequences $\{X_n\}_{n=2}^{\infty}$ and $\{Y_n\}_{n=2}^{\infty}$ converge in distribution to $\bar{X}$ and $\bar{Y}$, respectively. Since, by assumption, $Y_n \leq_{st} X_n$ for all $n \geq 2$, it follows from the closure property of the usual stochastic order (reported in Section 2) that $\bar{X} \geq_{st} \bar{Y}$, as was to be proven.

Next, suppose that $\{Y_n\}_{n=1}^{\infty}$converges in distribution to a random variable, say $\bar{Y}$. In this case, the conclusion that $\bar{X} \geq_{st} \bar{Y}$ follows immediately from the last three lines of the proof above, and the proof is finished. □

Theorem 1. *Let $\succeq$ be a reflexive and transitive binary relation, defined on the set of sequences of random variables, which satisfies the equity principle. Let $\{X_n\}_{n=1}^{\infty}$ be a (status-quo) sequence of random variables satisfying Assumption 1. Then, there exists another sequence of random variables $\{Y_n\}_{n=1}^{\infty}$ which satisfies the following properties:*

$$(i)\ (Y_1, Y_2, X_3, X_4, X_5, ...) \succeq \{X_n\}_{n=1}^{\infty}, X_1 \leq_{cv} Y_1, Y_1 \leq_{cv} Y_2, \textit{and}\ Y_2 \leq_{st} X_2;$$

(ii) for every $T \geq 3$, $(Y_1, Y_2, ...Y_T, X_{T+1}, X_{T+2}, X_{T+3}, ...)$ is obtained from $(Y_1, Y_2, ...Y_{T-1}, X_T, X_{T+1}, X_{T+2}, ...)$ applying an equity-improving transfer (as per equity principle) to Y_{T-1} and X_T, all else being equal,

$$(Y_1, Y_2, ...Y_T, X_{T+1}, X_{T+2}, X_{T+3}, ...) \succeq \{X_n\}_{n=1}^{\infty}, Y_n \leq_{cv} Y_{n+1}$$

for $n = 1, 2, ..T-1$, and $Y_n \leq_{st} X_n$ for $n = 2, 3, ...T$;

(iii) *if either the moment-generating function of* X_1 *exists on a neighborhood of zero* $(-\delta,\delta)$, *or* $\{Y_n\}_{n=1}^{\infty}$*converges in distribution,* $\{Y_n\}_{n=1}^{\infty}$*converges in distribution to* $\bar{Y}$ *and* $\bar{X} \geq_{st} \bar{Y}$.

Proof. Assume that $\succeq$ is a reflexive and transitive preference relation (on the set of sequences of random variables) satisfying the equity principle, and that $\{X_n\}_{n=1}^{\infty}$ is a sequence of random variables which satisfies Assumption 1. By Lemma 1 it will suffice to show the existence of another sequence of random variables $\{Y_n\}_{n=1}^{\infty}$ which satisfies properties (i) and (ii) in the statement of the theorem. To this end, I will first prove that property (i) holds, and then the proof of property (ii) will be performed by induction on T.
(i): By Assumption 1, $X_2 \geq_{st} X_1$, which implies $X_2 \geq_{icv} X_1$ (see Section 2). Therefore, by (7) there exists a random variable Z such that

$$X_1 \leq_{cv} Z \leq_{st} X_2. \tag{14}$$

In turn, the previous condition implies

$$X_1 \leq_{icv} Z \leq_{icv} Z \leq_{icv} X_2 \tag{15}$$

(see Section 2). Define the sequence $(Y_1,Y_2,X_3,X_4,X_5,...) := (Z,Z,X_3,X_4,X_5,...)$, and note that by (14), (15), and Definition 1, $(Z,Z,X_3,X_4,X_5,...)$ satisfies the desired properties, i.e.,

$$Y_1 = Z \geq_{cv} X_1, Y_1 = Z \leq_{cv} Y_2 = Z, Y_2 = Z \leq_{st} X_2,$$

and $(Z,Z,X_3,X_4,X_5,...) \succeq \{X_n\}_{n=1}^{\infty}$.
(ii):
Step 1 of the proof by induction ($T=3$): By (14), $Z \leq_{st} X_2$. On the other hand, by Assumption 1, $X_2 \leq_{st} X_3$. Therefore, transitivity of the usual stochastic order yields $Z \leq_{st} X_3$, which implies (see Section 2)

$$Z \leq_{icv} X_3. \tag{16}$$

Therefore, by (7), there exists a random variable Z_1 such that

$$Z \leq_{cv} Z_1 \leq_{st} X_3. \tag{17}$$

Since the usual stochastic order and the concave order both imply the increasing concave order (see Section 2), (17) implies

$$Z \leq_{icv} Z \leq_{icv} Z_1 \leq_{icv} X_3. \tag{18}$$

Now, define the sequence $(Y_1,Y_2,Y_3,X_4,X_5,X_6,...) := (Z,Z,Z_1,X_4,X_5,X_6,...)$. Clearly, in view of (18) and Definition 1, $(Y_1,Y_2,Y_3,X_4,X_5,X_6,...)$ is obtained from

$$(Y_1,Y_2,X_3,X_4,X_5,...) := (Z,Z,X_3,X_4,X_5,...)$$

by applying an equity-improving transfer to Y_2 and X_3, all ese being equal. Also, it follows from (18) and Definition 1 that $(Z,Z,Z_1,X_4,X_5,X_6,...) \succeq (Z,Z,X_3,X_4,X_5,...)$. Hence, the above proof of (i) and transitivity of the preference relation yields $(Z,Z,Z_1,X_4,X_5,X_6,...) \succeq \{X_n\}_{n=1}^{\infty}$. To finish the proof of Step 1, observe that $Y_n \leq_{cv} Y_{n+1}$, for $n = 1,2$, holds true as a consequence of (17) and, of course, $Z \leq_{cv} Z$. Additionally, $Y_n \leq_{st} X_n$, for $n = 2,3$, holds true as a consequence of (14) and (17).
Step 2 of the proof by induction (Induction step):

Assume that $(Y_1,Y_2,...Y_T,X_{T+1},X_{T+2},X_{T+3},...)$ is obtained from

$$(Y_1,Y_2,...Y_{T-1},X_T,X_{T+1},X_{T+2},...)$$

applying an equity-improving transfer (as per equity principle) to Y_{T-1} and X_T, all else being equal, that $(Y_1, Y_2, ...Y_T, X_{T+1}, X_{T+2}, X_{T+3}, ...) \succeq \{X_n\}_{n=1}^{\infty}$, and that

$$Y_n \leq_{cv} Y_{n+1} \text{ for } n = 1, 2, ..T-1, \text{and } Y_n \leq_{st} X_n \text{ for } n = 2, 3, ...T.$$

I must show that there exists a sequence of random variables

$$(Y_1, Y_2, ...Y_{T+1}, X_{T+2}, X_{T+3}, X_{T+4}, ...)$$

which satisfies the analogous properties. To begin the proof, note that by assumption $Y_T \leq_{st} X_T$. Also, by Assumption 1 $X_T \leq_{st} X_{T+1}$. Thus, transitivity of the usual stochastic order implies $Y_T \leq_{st} X_{T+1}$, which in turn implies (see Section 2) $Y_T \leq_{icv} X_{T+1}$. Therefore, (7) guarantees the existence of a random variable Θ such that

$$Y_T \leq_{cv} \Theta \leq_{st} X_{T+1}. \tag{19}$$

Since the usual stochastic order and the concave order both imply the increasing concave order (see Section 2), (19) easily implies

$$Y_T \leq_{icv} Y_T \leq_{icv} \Theta \leq_{icv} X_{T+1}. \tag{20}$$

Next, define the new sequence

$$(Y_1, Y_2, ...Y_{T+1}, X_{T+2}, X_{T+3}, X_{T+4}, ...) := (Y_1, Y_2, ...Y_{T-1}, Y_T, \Theta, X_{T+2}, X_{T+3}...).$$

It should be clear from (20) that this new sequence is obtained from the given $(Y_1, Y_2, ...Y_T, X_{T+1}, X_{T+2}, X_{T+3}, ...)$ applying an equity-improving transfer (as per equity principle) to Y_T and X_{T+1}, all else being equal. Therefore, by Definition 1

$$(Y_1, Y_2, ...Y_{T-1}, Y_T, \Theta, X_{T+2}, X_{T+3}...) \succeq (Y_1, Y_2, ...Y_T, X_{T+1}, X_{T+2}, X_{T+3}, ...).$$

By the given assumption of the induction step, the latter sequence is preferred to the status-quo sequence. Therefore, transitivity of the preference relation yields $(Y_1, Y_2, ...Y_{T-1}, Y_T, \Theta, X_{T+2}, X_{T+3}...) \succeq \{X_n\}_{n=1}^{\infty}$. Moreover, recall that by assumption $Y_{T-1} \leq_{cv} Y_T$. Also, it follows from (19) that

$$Y_T \leq_{cv} \Theta = Y_{T+1}. \tag{21}$$

Furthermore, by construction of the new sequence and by (19) the following holds:

$$Y_{T+1} = \Theta \leq_{st} X_{T+1}. \tag{22}$$

(21) and (22) show that the newly-defined sequence satisfies all the desired properties. The proof of the induction step is now complete. □

Remark 4. The proof by induction above leads to the following interpretation of Theorem 1: looking at the proof of parts (i) and (ii), one can imagine a policy-maker with a two-period planning horizon (i.e., a short-sighted policy-maker). This policy-maker emphasizes equity over other relevant principles. In an effort to boost equity, at every date the policy-maker may or may not change the current income distribution (depending on whether said distribution turns out to be different from the one that had been planned one period before) and also implements a policy to affect the distribution of income one period ahead in the future. This is done applying an equity-improving transfer (like the one displayed in Definition 1). This series of policy interventions makes the policy-maker weakly better-off (at every step of this process) and also improves upon the status-quo sequence $\{X_n\}_{n=1}^{\infty}$, according to preferences satisfying the equity principle. This process continues, moving forward, and eventually the policy-maker ends up with a new sequence of income distributions $\{Y_n\}_{n=1}^{\infty}$. In this sense, equity goes up when transitioning from $\{X_n\}_{n=1}^{\infty}$ to $\{Y_n\}_{n=1}^{\infty}$. However, part *(iii)* and the concept of equity-efficiency trade-off in the long run (explained at the end of Section 3) suggest that: since $\bar{X} \geq_{st} \bar{Y}$, if the asymptotic income distributions $\bar{X}$ and $\bar{Y}$ exist, the policy-maker's piecemeal pursuit of equity may undermine efficiency in the long-run (i.e., it may come at the expense of income of future generations).

Below is an example of transitive pereferences (a strengthening of Rawlsian preferences) that satisfy the equity principle. I will use it to better clarify the scope of Theorem 1 in Remark 5: restrict attention to the set of sequences of random variables such that both the inf and the sup of the sequence of the corresponding expected values are well-defined. It is easy to check that $\{X_n\}_{n=1}^{\infty}$ and $\{Y_n\}_{n=1}^{\infty}$, featured in Theorem 1, both satisfy this property. Consider the preference relation $\succeq$ defined by: $\{A_n\}_{n=1}^{\infty} \succeq \{B_n\}_{n=1}^{\infty}$ if and only if

$$\begin{aligned}
&\inf\left(\{E\left[A_n\right]\}_{n=1}^{\infty}\right) > \inf\left(\{E\left[B_n\right]\}_{n=1}^{\infty}\right) \text{ or}\\
&[\inf\left(\{E\left[A_n\right]\}_{n=1}^{\infty}\right) = \inf\left(\{E\left[B_n\right]\}_{n=1}^{\infty}\right)\\
&\text{and } \sup\left(\{E\left[A_n\right]\}_{n=1}^{\infty}\right) \leq \sup\left(\{E\left[B_n\right]\}_{n=1}^{\infty}\right)].
\end{aligned}$$

It is time-consuming but easy to prove that this preference relation is transitive and satisfies the equity principle. It does not satisfy monotonicity, though.

Remark 5. Refer to Theorem 1. Suppose that the policy-maker is endowed with a strengthening of Rawlsian preferences (i.e., the preferences constructed above). Using Assumption 1, (1), (4), and the proof of Theorem 1, it is not difficult to show that each step along the transition from the status-quo sequence $\{X_n\}_{n=1}^{\infty}$ to the new sequence $\{Y_n\}_{n=1}^{\infty}$ gives rise to a sequence of random variables which is weakly preferred to the sequence from the previous step. Furthermore, it turns out that $\{Y_n\}_{n=1}^{\infty} \succeq \{X_n\}_{n=1}^{\infty}$. Therefore, equity goes up along the transition and the new sequence reached by the policy-maker is (weakly) more equitable than the status-quo. On the other hand, as the proof of Theorem 1 shows, in the long-run the policy-maker may sacrifice efficiency, which is the price to pay to boost equity.

5. ILLUSTRATIVE EXAMPLES

I will sketch the construction of two examples that illustrate Theorem 1, omitting lengthy but easy-to-check details. All examples involve statistical distributions that are normally used to model income distributions.

Example 1. $P\left(x_m, \alpha\right)$ will denote the Pareto distribution with scale and shape parameters given, respectively, by x_m and α. Let the status-quo sequence be defined by

$$\{X_n\}_{n=1}^{\infty} = \left(X_1, X_2, X_2, X_2, X_2 \ldots\right),$$

where $X_1 \sim P\left(1,3\right)$ and $X_2 \sim P\left(2,3\right)$. Let $F_{X_i} : \mathbb{R} \to [0,1)$, with $i = 1,2$, be the cumulative distribution function of X_1 and X_2, respectively. It is easy to check that $F_{X1}\left(x\right) \geq F_{X2}\left(x\right)$ for all $x \in \mathbb{R}$. Therefore, it follows from the definition of the usual stochastic order (see [10, Section 1.A.1]) that $X_2 \geq_{st} X_1$. Of course, $\{X_n\}_{n=1}^{\infty}$converges in distribution to $X_2 \sim P\left(2,3\right)$ and is bounded and increasing in the usual stochastic order sense. Therefore, $\{X_n\}_{n=1}^{\infty}$ has the same properties as the status-quo sequence in Theorem 1. In what follows, I will show how to construct another sequence of random variables that has the same properties as the sequence $\{Y_n\}_{n=1}^{\infty}$ in Theorem 1. For the sake of simplicity I will exhibit a constant sequence, but, as should be clear from the proof of Theorem 1, the sequence $\{Y_n\}_{n=1}^{\infty}$ in Theorem 1 need not be constant. To begin, define the new random variable

$$Z = E[X_1 \mid X_2]. \tag{23}$$

Incidentally, one could assume that the joint density function of X_1 and X_2 is a bivariate Pareto distribution of the first kind, consistent with the marginals $P(1,3)$ and $P(2,3)$, and then write an expression for Z as a function of X_2. However, I deem that doing so would not add any significant insight to this example. I can assume, without loss of generality, that X_1 and X_2 are both defined on the same probability space[3]. Thus, by the law of iterated expectations $E[Z] = E[X_1] = \frac{3}{2}$. Since $X_2 \geq_{st} X_1$ implies $X_2 \geq_{icv} X_1$, it follows from [10, Theorem 4.A.5] that

$$Z \leq X_2 \text{ almost surely.} \tag{24}$$

By [10, Theorem 4.A.5], (24) implies

$$X_2 \geq_{st} Z. \tag{25}$$

Note that (23) implies $E[X_1 \mid Z] = Z$ almost surely. Therefore, it follows from [10, Theorem 3.A.4] that $X_1 \geq_{cx} Z$, which is equivalent to

$$X_1 \leq_{cv} Z \tag{26}$$

(see (3) above). Define the constant sequence $\{Y_n\}_{n=1}^{\infty} = (Z,Z,Z,Z,Z...)$. Note that (25) and (26) imply

$$\begin{gathered} X_1 \leq_{icv} Z \leq_{icv} Z \leq_{icv} X_2 \text{ and} \\ Z \leq_{icv} Z \leq_{icv} Z \leq_{icv} X_2. \end{gathered} \tag{27}$$

Moreover, $\{X_n\}_{n=1}^{\infty}$ converges in distribution to X_2 and $\{Y_n\}_{n=1}^{\infty}$ converges in distribution to Z, where $X_2 \geq_{st} Z$. Therefore, it is easy to verify that $(Z,Z,Z,Z,Z...)$ satisfies the same properties as the sequence $\{Y_n\}_{n=1}^{\infty}$ in Thereom 1. More specifically, using Definition 1, (25), (26) and (27), it is easy to check that

$$(Z,Z,X_2,X_2...) \succeq \{X_n\}_{n=1}^{\infty},\ (Z,Z,Z,X_2,X_2...) \succeq (Z,Z,X_2,X_2...), \text{ etc.,}$$

and that all the other properties stated in Theorem 1 are satisfied. Finally, note that since $X_2 \geq_{st} Z$, $\{X_n\}_{n=1}^{\infty}$ and $(Z,Z,Z,Z,Z...)$ can be ranked according to the long-run efficiency criterion but they are not comparable according to the monotonicity principle. This is because, in view of (26) above, $X_1 \geq_{st} Y_1$ does not hold despite $X_n \geq_{st} Y_n$ for all $n \geq 2$.

The second example (presented below) will hinge on a result I establish next. This result may be of some interest in its own right: it showcases how well the definition of moment-generating function fits with the definition of stochastic orders, and shows that well-known properties of monotonic and bounded sequences of real numbers carry over to sequences of random variables (whose moment generating functions exist).

Proposition 1. *Let* $\{T_n\}_{n=1}^{\infty}$ *be a sequence of random variables satisfying the following properties:* (i) $T_{n+1} \geq_{st} T_n$ *for all* $n \geq 1$, *and there exists a random variable* $\hat{T}$ *such that* $\hat{T} \geq_{st} T_n$ *for all* $n \geq 1$. (ii) *The moment-generating functions* $M_{\hat{T}} : \mathbb{R} \to \mathbb{R}$ *and* $M_{T_n} : \mathbb{R} \to \mathbb{R}$, *for all* $n \geq 1$, *exist on some common open neighborhood of* 0, *say* $(-\delta, \delta)$. *Then,* $\{T_n\}_{n=1}^{\infty}$ *converges in distribution to a unique random variable* $\bar{T}$.

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[3] I.e., $(0,1)$ equipped with the $\sigma-$algebra of Lebesgue measurable subsets and the Lebesgue measure on the real line.

Proof. Given any $t \in \mathbb{R}$, let $f : \mathbb{R} \to \mathbb{R}$ be the function defined by $f(x) = e^{tx}$. Recall, from Section 2, that $Y \geq_{st} X$ implies $Y \geq_{icx} X$. So, it follows from the stated assumption that

$$T_{n+1} \geq_{icx} T_n \text{ and } \hat{T} \geq_{icx} T_n \text{ for all } n \geq 1. \tag{28}$$

Suppose that $t \in (0, \delta)$. Clearly, f is nondecreasing and convex (as a function of x). Thus, by (6) and (28), and using the definition of moment-generating function (which is well-defined, by assumption),

$$M_{T_{n+1}}(t) \geq M_{T_n}(t), \text{ and } M_{\hat{T}}(t) \geq M_{T_n}(t) \text{ for all } n \geq 1. \tag{29}$$

It should be clear that (29) implies that $\{M_{T_n}\}_{n=1}^{\infty}$ converges pointwise on $(0, \delta)$. Next, suppose that $t \in (-\delta, 0)$. Clearly, f is nonincreasing and convex (as a function of x). So, $-f$ is nondecreasing and concave as a function of x. Recall, from Section 2, that $Y \geq_{st} X$ implies $Y \geq_{icv} X$. So, it follows from the stated assumption that

$$T_{n+1} \geq_{icv} T_n \text{ and } \hat{T} \geq_{icv} T_n \text{ for all } n \geq 1. \tag{30}$$

Therefore, it follows from (6), (30), and the definition of moment-generating function (which is assumed to exist) that

$$M_{T_{n+1}}(t) \leq M_{T_n}(t), \text{ and } M_{\hat{T}}(t) \leq M_{T_n}(t) \text{ for all } n \geq 1. \tag{31}$$

(31) easily implies that $\{M_{T_n}\}_{n=1}^{\infty}$ converges pointwise on $(-\delta, 0)$. Finally, suppose that $t = 0$. In this case, by definition of moment-generating function $\{M_{T_n}(0)\}_{n=1}^{\infty}$ is the constant sequence $\{1, 1, 1, ...\}$. Thus, putting together the three cases just analyzed, $\{M_{T_n}\}_{n=1}^{\infty}$ converges pointwise on $(-\delta, \delta)$. Therefore, by Curtiss's convergence theorem (see the end of Section 2) $\{T_n\}_{n=1}^{\infty}$ converges in distribution to a unique random variable $\bar{T}$. □

Example 2. This example deals with log-normal distributions.

Let $T_n \sim N(\mu_n, \sigma^2)$ for all $n \geq 1$, and let $\hat{T} \sim N(\hat{\mu}, \sigma^2)$. Assume that $\mu_n \leq \mu_{n+1}$ and $\mu_n \leq \hat{\mu}$ for all $n \geq 1$. Then, $T_{n+1} \geq_{st} T_n$ and $\hat{T} \geq_{st} T_n$ for all $n \geq 1$ (see [10, Example 1.A.26]). Therefore, by Proposition 1 above the sequence $\{T_n\}_{n=1}^{\infty}$ converges in distribution to a random variable, say $\bar{T}$.

Next, define the status-quo sequence $\{X_n\}_{n=1}^{\infty}$ as follows: $X_n = e^{T_n}$ for all $n \geq 1$. Also, let $\hat{X} = e^{\hat{T}}$. The function $f : \mathbb{R} \to \mathbb{R}$, defined by $f(x) = e^x$, is increasing and the usual stochastic order is closed under increasing transformations (see [10, Theorem 1.A.3., part (a)]). Thus $X_{n+1} \geq_{st} X_n$ and $\hat{X} \geq_{st} X_n$ for all $n \geq 1$. The foregoing function is also continuous. Therefore, by Mann and Wald's continuous mapping theorem, $f(T_n) = X_n \xrightarrow{d} f(\bar{T}) = e^{\bar{T}}$, where $\xrightarrow{d}$ denotes convergence in distribution. Thus, convergence in distribution obtains, and the sequence $\{X_n\}_{n=1}^{\infty}$ satisfies the same properties as the status-quo sequence in Theorem 1. Parenthetically, using standard properties of the log-normal distribution it is easy to check that the assumptions on $\{T_n\}_{n=1}^{\infty}$ (i.e., $\mu_n \leq \mu_{n+1}$ and constant variance σ^2) lead to a status-quo sequence of income distributions $\{X_n\}_{n=1}^{\infty}$ that features increasing mean and variance. Therefore, this example may be viewed also as a framework for an economy with widening business cycle fluctuations around the upward trend of the real gross domestic product per capita (in the absence of economic policy). Regarding a new sequence that has the same properties as the sequence $\{Y_n\}_{n=1}^{\infty}$ in Theorem 1, define, recursively, the sequence

$$\{Y_n\}_{n=1}^{\infty} = (Z, Z, Z_1, Z_2, Z_2, Z_2, Z_2 ...), \tag{32}$$

where $Z = E[X_1 \mid X_2]$, $Z_1 = E[Z \mid X_3]$, and $Z_2 = E[Z_1 \mid X_4]$. Using the same theorems (from [10]) as the ones invoked for Example 1, in conjunction with Definition 1, it is not difficult to verify that

$$\begin{aligned}(Z,Z,X_3,X_4...) &\succeq \{X_n\}_{n=1}^{\infty},\ (Z,Z,Z_1,X_4,X_5...) \succeq (Z,Z,X_3,X_4...),\\ (Z,Z,Z_1,Z_2,X_5...) &\succeq (Z,Z,Z_1,X_4,X_5...),\ \text{etc.,}\end{aligned}$$

and that the sequence (32) satisfies the same properties as the sequence $\{Y_n\}_{n=1}^{\infty}$ in Theorem 1.

6. CONCLUSIONS

The focus of this paper was on building a tractable model for analyzing the potential trade-off between long-run efficiency and equity when the policy maker is short-sighted and takes a piecemeal approach to improving equity of income distributions over time. I believe that this work also showcases the types of research questions that could be addressed and results which could be pursued by using the theory of stochastic orders in a full fledged fashion. One open research question, which I have not settled and is left for future research, is whether the assumption that the status-quo sequence is bounded from above can be dropped or relaxed. Another issue worth exploring, which is left for future research, is whether it is possible to tackle the concepts of long run and convergence with other methods and assumptions (e.g., Markov chains and invariant distributions).

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