CONTROL STRUCTURES AND CONTROL ALGORITHMS: THE ADVANTAGES OF YOULA PARAMETERIZATION

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Dedicated to the memory of Professor Josef Shinar on the occasion of his 90th birthday
In memoriam Professor Dante C. Youla who passed away in 2021.

Abstract. Systems have to be controlled to ensure the required properties both in steady state and during the transient response. Control systems are based on negative feedback. The output of the system is compared to the reference signal and the difference serves as the input of the controller unit. The output of the controller modifies the input of the process. The structure and the parameters of the controller have to be determined appropriately to ensure the requested control performance. The basic feedback structure ensures reference signal tracking and disturbance rejection as well. Several extensions of the usual control structure are given as using not only the output signal for feedback but also other available information (inner signals, measurable disturbance) the performance can be improved. Nowadays newer control paradigms have come into the front of interest both relating control structures enhancing the opportunities of negative feedback, and control algorithms which can handle better e.g., big dead time, uncertainties, nonlinearities in the system to be controlled. YOULA-parameterization provides simple design and effective control even for systems containing big dead time. The paper gives an overview, description, evaluation and comparison of the different control paradigms. It is shown that the best usable method is the YOULA-parameterization based regulator design and the so-called KB parameterization introduced by the authors. Several examples illustrate the performance of the control systems.
Keywords. Control performance; Control structures; Control algorithms; Regulator design; YOULA parameterization.
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1. INTRODUCTION

Systems are all around us. Systems can be physical, chemical, biological, medical, environmental, social, etc. systems. Behaviour of systems has to be understood, analysed, controlled. The inputs of a system are the signals coming from the environment and acting on the system. These can be actuating signals which can be manipulated, or disturbances. The input signals acting on the system modify the system output.

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The behaviour of the system can be analysed based on its model which describes mathematically the relationship between the system inputs and outputs. The model can be built understanding and describing the physical operation of the system by mathematical equations and determining the parameters appearing in the equations by measurements or by identification, when a structure of the model equation is supposed, and exciting the system with appropriate input signals the output is measured, and the parameters of the model are calculated by identification algorithms.

The desired behaviour of the system is given by specifications. The specifications prescribe the static and dynamic performance of the system. Control means a specific action modifying the system input to reach the desired output performance. The main control structure is based on negative feedback, which ensures reference signal tracking and disturbance rejection as well. Control is realized by connecting elements of special tasks (measuring signals, comparing the reference and the measured output signal, executing modification of the error signal with a controller unit, etc.) to the system.

The control system has to be not very sensitive to measurement noises and to plant/model mismatch. The designed control system has to ensure various quality specifications. Also, it has to be technically realizable and eligible to economical and other (e.g., environmental protection or safety) viewpoints.

The requirements set for a control system are:

- stability
- appropriate static accuracy for reference signal tracking and disturbance rejection
- attenuation of the effect of measurement noise
- robustness to parameter changes
- prescribed dynamic (transient) behaviour
- consideration of the restrictions due to practical realization
- consideration of nonlinearities of the system model

There are several structural extensions of the simple feedback control system, which may improve reference signal tracking and disturbance rejection properties. The parameters of the controller are designed using control algorithms ensuring the required performance.

The paper gives description, evaluation and comparison of the different control paradigms. It is shown that the best usable control method is the YOULA-parameterization based regulator. The KB parameterization method introduced by the first two authors is also presented. Several examples illustrate the performance of the control systems.

2. MODELS OF SYSTEMS

The model of the system gives the mathematical relationship between the inputs and the outputs of the system and may consider also the inner, so called state variables.

Let us consider linear time invariant (LTI) continuous-time (CT) single input–single output (SISO) systems.

The LTI state-space equations of a system can be given as

$$\begin{aligned} \frac{\mathrm{d}\boldsymbol{x}(t)}{\mathrm{d}t} &= \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{b}u(t), \\ y(t) &= \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}(t) + d_{\mathrm{c}}u(t). \end{aligned} \tag{1}$$

Here u and y are the input and output signals of the process, respectively, and $\boldsymbol{x}$ is the state vector. The parameter matrices of the system are $\boldsymbol{A}, \boldsymbol{b}, \boldsymbol{c}^{\mathrm{T}}, d$. In n-order case, matrix $\boldsymbol{A}$ means a square matrix of $(n \times n)$ dimension, which is the so-called state matrix, $\boldsymbol{b}$ is a column vector of $(n \times 1)$ size, $\boldsymbol{c}^{\mathrm{T}}$ is a row vector of $(1 \times n)$ size, and d_{c} is scalar.

The classical model of the dynamic LTI processes, the transfer function $P(s)$ is defined by the ratio of the LAPLACE transforms of the output and the input signals, respectively, which can be easily derived from the state equation (1).

$$P(s) = \frac{Y(s)}{U(s)} = \boldsymbol{c}^{\mathrm{T}}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b} + d_c = \frac{\mathscr{B}(s)}{\mathscr{A}(s)}, \tag{2}$$

where

$$\begin{aligned} \mathscr{A}(s) &= \det(s\boldsymbol{I} - \boldsymbol{A}) = s^n + a_1 s^{n-1} + \ldots + a_n, \\ \mathscr{B}(s) &= b_0 s^m + b_1 s^{m-1} + \ldots + b_m. \end{aligned} \tag{3}$$

The roots of equation $\mathscr{A}(s) = 0$ are called poles; the roots of $\mathscr{B}(s) = 0$ are called zeros. A CT linear process is stable, if all roots of the polynomial $\mathscr{A}(s)$ are located on the left-hand side of the complex plane. Concerning the order of the polynomials $\mathscr{A}(s)$ and $\mathscr{B}(s)$ it should be noted that the number of the state variables is n, m is the order of the polynomial $\mathscr{B}(s)$, and for physically realizable systems the relation $m \leq n$ holds. The difference between the order of the numerator and the denominator $p_{\mathrm{T}} = n - m$ is called pole access. If $p_{\mathrm{T}} > 0$ then $P(s)$ is strictly proper, if $p_{\mathrm{T}} = 0$ then the transfer function is proper. In the practice arbitrary relation $0 \leq p_{\mathrm{T}} \leq n$ might occur.

Control algorithms are designed considering the model of the system. If the system is slightly nonlinear, it can be linearized around working points, and for small variations around the working point linear control algorithms can be applied. Of course the parameters of the algorithms will be different in the different working points.

3. Basic Control Structures and Control Algorithms

Control systems are based on feedback. If the model of the system is given by state equations, the control can be realized by state feedback. In this case the inner variables, the so-called state variables should be measured or estimated and fed back to the input. If the model is the input-output relationship given by the transfer function, then output feedback can be realized.

Nowadays control systems are realized more and more frequently by computer control, where the measured variables are sampled and discretized, then connected to the process control computer. The computer calculates the control signal in real time, in every sampling time and through digital/analog converter forwards the control signal to the system input. Thus continuous or discrete control algorithms are applied ([1, 3, 4, 8, 9, 10]).

3.1. **Control loops with state feedback.** It was shown formerly how processes are represented in state-space. In many cases this model is available only and the transfer function of the controlled system is unavailable. This partly explains why control design methodology directly based on state-space description has been evolved. Let us consider the state-space representation of a LTI process to be controlled such as

$$\begin{aligned} \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} &= \dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{b}u, \\ y &= \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x}, \end{aligned} \tag{4}$$

which corresponds to (1) for the case of $d_c = 0$. This does not violate the generality, because it is very rare that the model contains a proportional channel directly affecting the output. The block scheme of (4) and the classical state-feedback is shown in Fig. 1, where the thick lines present vector variables and r denotes the reference signal.

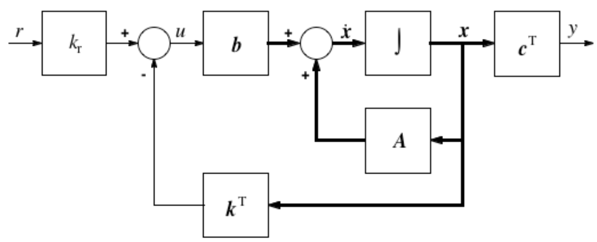

FIGURE 1. Linear regulator with state feedback

In the closed-loop the state vector is fed back with the linear proportional vector $\boldsymbol{k}^{\mathrm{T}}$ according to the expression

$$u = k_{\mathrm{r}}r - \boldsymbol{k}^{\mathrm{T}}\boldsymbol{x} \tag{5}$$

Based on Fig. 1, the state equation of the closed-loop system can be easily written as

$$\begin{aligned} \frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} &= \left(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}^{\mathrm{T}}\right)\boldsymbol{x} + k_{\mathrm{r}}\boldsymbol{b}r \\ y &= \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} \end{aligned} \tag{6}$$

i.e., with the state feedback the dynamics represented by the original system matrix $\boldsymbol{A}$ is modified by the dyadic product $\boldsymbol{b}\boldsymbol{k}^{T}$ to $(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}^{\mathrm{T}})$.

The transfer function of the closed-loop control system is

$$\begin{aligned} T_{ry}(s) &= \frac{Y(s)}{R(s)} = \boldsymbol{c}^{\mathrm{T}}(s\boldsymbol{I} - \boldsymbol{A} + \boldsymbol{b}\boldsymbol{k}^{\mathrm{T}})^{-1}\boldsymbol{b}k_r = \\ &= \frac{\boldsymbol{c}^{\mathrm{T}}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}k_r}{1 + \boldsymbol{k}^{\mathrm{T}}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}} = \frac{k_r}{1 + \boldsymbol{k}^{\mathrm{T}}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}}P(s) = \frac{k_r\mathscr{B}(s)}{\mathscr{A}(s) + \boldsymbol{k}^{\mathrm{T}}\boldsymbol{\Psi}(s)\boldsymbol{b}} \end{aligned} \tag{7}$$

which is derived from the comparison of equations valid for the LAPLACE transforms, $U(s) = k_{\mathrm{r}}R(s) - \boldsymbol{k}^{\mathrm{T}}\boldsymbol{X}(s)$ (see [6]) and $Y(s) = \boldsymbol{c}^{\mathrm{T}}\boldsymbol{X}(s)$ (see [4]) using the matrix inversion lemma. Note that the state feedback leaves the zeros of the process untouched and only the poles of the closed-loop system can be designed by $\boldsymbol{k}^{\mathrm{T}}$.

The calibration factor k_{r} is introduced in order to make the gain of T_{ry} equal to unity $\big(T_{\mathrm{ry}}(0) = 1\big)$. The open loop is obviously not of type one, so it cannot provide zero error and unity static transfer gain. It can be ensured only if the condition

$$k_r = \frac{-1}{\boldsymbol{c}^{\mathrm{T}}(\boldsymbol{A} - \boldsymbol{b}\boldsymbol{k}^{\mathrm{T}})^{-1}\boldsymbol{b}} = \frac{\boldsymbol{k}^{\mathrm{T}}\boldsymbol{A}^{-1}\boldsymbol{b} - 1}{\boldsymbol{c}^{\mathrm{T}}\boldsymbol{A}^{-1}\boldsymbol{b}} \tag{8}$$

is fulfilled. The above special control loop is called state feedback.

Pole placement by state feedback.

The most natural design method of state feedback is the so-called pole placement. In this case the feedback vector $\boldsymbol{k}^{\mathrm{T}}$ needs to be chosen to make the characteristic equation of the closed-loop equal to the prescribed, so-called design polynomial $\mathscr{R}(s)$, i.e.,

$$\mathscr{R}(s)=s^n+r_1s^{n-1}+\ldots+r_{n-1}s+r_n=\det(s\boldsymbol{I}-\boldsymbol{A}+\boldsymbol{b}\boldsymbol{k}^{\mathrm{T}})=\mathscr{A}(s)+\boldsymbol{k}^{\mathrm{T}}\boldsymbol{\Psi}(s)\boldsymbol{b} \tag{9}$$

The solution always exists if the process is controllable. (It is reasonable if the order of $\mathscr{R}$ is equal to that of A.) In the exceptional case when the transfer function of the controlled system is known, the canonical state equations can be directly written. In the controllable canonical form the system matrices are

$$\boldsymbol{A}_c=\begin{bmatrix} -a_1 & -a_2 & \ldots & -a_{n-1} & -a_n \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{10}$$
$$\boldsymbol{c}_{\mathrm{c}}^{\mathrm{T}}=[b_1,b_2,\ldots,b_n]; \quad \boldsymbol{b}_{\mathrm{c}}=[1,0,\ldots,0]^{\mathrm{T}}.$$

Considering the special forms of $\boldsymbol{A}_c$ and $\boldsymbol{b}_{\mathrm{c}}$, it can be seen that the design equation (9) results in

$$\boldsymbol{k}^{\mathrm{T}}=\boldsymbol{k}_{\mathrm{c}}^{\mathrm{T}}=[r_1-a_1,r_2-a_2,\ldots,r_n-a_n] \tag{11}$$

ensuring the characteristic equation ($\mathscr{R}(s)=0$), i.e., the prescribed poles. The choice of the calibration factor can be determined by simple calculation

$$k_{\mathrm{r}}=\frac{a_n+(r_n-a_n)}{b_n}=\frac{r_n}{b_n}. \tag{12}$$

Based on equations (7), (8) and (9) it can be seen that in the case of state feedback pole placement the closed-loop transfer function results in

$$T_{\mathrm{ry}}(s)=\frac{k_{\mathrm{r}}\mathscr{B}(s)}{\mathscr{R}(s)}. \tag{13}$$

The most common case of state feedback is when not the transfer function but the state-space form of the control system is given. It has to be observed that all controllable systems can be described in a controllable canonical form by using the transformation matrix $\boldsymbol{T}_{\mathrm{c}}=\boldsymbol{M}_{\mathrm{c}}^{\mathrm{c}}(\boldsymbol{M}_{\mathrm{c}})^{-1}$, where $\boldsymbol{M}_{\mathrm{c}}=[\ \boldsymbol{b}\quad \boldsymbol{A}\boldsymbol{b}\quad \ldots\quad \boldsymbol{A}^{n-1}\boldsymbol{b}\ ]$ is the controllability matrix and $\boldsymbol{M}_{\mathrm{c}}^{\mathrm{c}}$ is the controllability matrix of the controllable canonical form.

This linear transformation also refers to the feedback vector

$$\begin{aligned} \boldsymbol{k}^{\mathrm{T}}&=\boldsymbol{k}_{\mathrm{c}}^{\mathrm{T}}\boldsymbol{T}_{\mathrm{c}}=\boldsymbol{k}_{\mathrm{c}}^{\mathrm{T}}\boldsymbol{M}_{\mathrm{c}}^{\mathrm{c}}\boldsymbol{M}_{\mathrm{c}}^{-1} \\ \boldsymbol{k}^{\mathrm{T}}&=\boldsymbol{b}_{\mathrm{c}}^{\mathrm{T}}\boldsymbol{M}_{\mathrm{c}}^{-1}\mathscr{R}(\boldsymbol{A})=[0,0,\ldots,1]\boldsymbol{M}_{\mathrm{c}}^{-1}\mathscr{R}(\boldsymbol{A}). \end{aligned} \tag{14}$$

The design relating to the controllable canonical form (10), together with the linear transformation relationship corresponding to the first row of the non-controllable form (14), is known as the BASS-GURA algorithm. The algorithm in the second row of (14) is called ACKERMANN method after its elaborator.

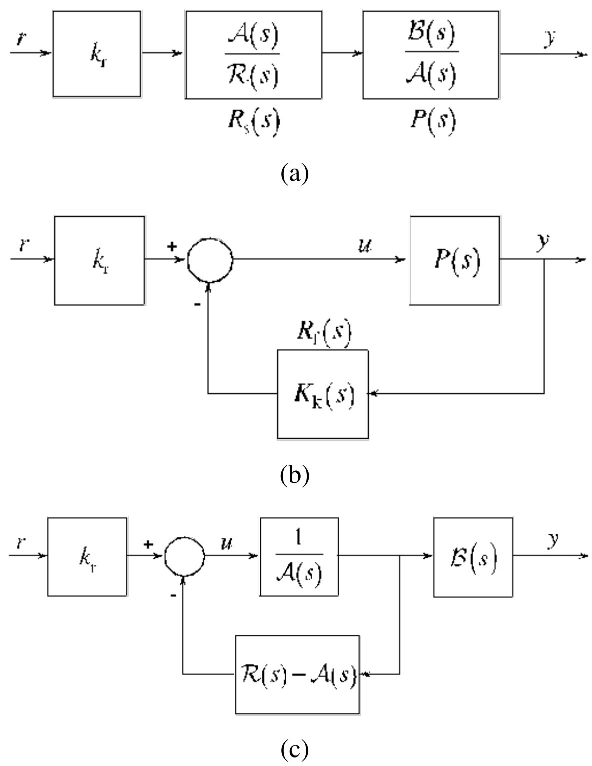

FIGURE 2. Equivalent schemes of the state feedback design using transfer functions and polynomials

In the BASS-GURA algorithm, the inverse of the controllability matrix $\boldsymbol{M}_{\mathrm{c}}$ needs to be determined by the general system matrices $\boldsymbol{A}$ and $\boldsymbol{b}$ on the one hand and the controllability matrix $\boldsymbol{M}_{\mathrm{c}}^{\mathrm{c}}$ of the controllable canonical form, on the other. Since this latter term depends only on the coefficients a_i in the denominator of the process transfer function, the denominator needs to be calculated: $A(s) = \det(s\boldsymbol{I} - \boldsymbol{A})$. Since $[0, 0, \ldots, 1]\boldsymbol{M}_{\mathrm{c}}^{-1}$ is the last row of the inverse of the controllability matrix, and $\mathscr{R}(\boldsymbol{A})$ also needs to be calculated; the ACKERMANN method does not need the calculation of $\mathscr{A}(s)$.

It is worth mentioning that the state feedback formally corresponds to a conventional *PD* control and therefore over-actuating peaks are expected at the input of the process because the differentiating element tries to make the process faster. In practice, however, the actuator usually limits the amplitude of the peaks. This has to be taken into account during the design of the poles of the characteristic polynomial $\mathscr{R}(s)$.

It can be clearly seen that state feedback formally corresponds to a serial compensation $R_{\mathrm{s}} = k_{\mathrm{r}}\mathscr{A}(s)/\mathscr{R}(s)$ (Fig. 2(a)). The real operation and effect of the state feedback can be easily understood by the equivalent block schemes using the transfer functions shown in Fig. 2. The"regulator" $R_{\mathrm{f}}(s)$ of the closed-loop is in the feedback line (Fig. 2(b)). The transfer function of the closed-loop is

$$T_{\mathrm{ry}}(s) = \frac{k_{\mathrm{r}}\mathscr{B}(s)}{\mathscr{R}(s)} = \frac{k_{\mathrm{r}}\mathscr{B}(s)}{\mathscr{A}(s) + \mathscr{B}(s)} = \frac{k_{\mathrm{r}}P(s)}{1 + K_{\mathrm{k}}(s)P(s)} = \frac{k_{\mathrm{r}}\mathscr{A}(s)}{\mathscr{R}(s)}\frac{\mathscr{B}(s)}{\mathscr{A}(s)} = k_{\mathrm{r}}R_s(s)P(s), \qquad (15)$$

where

$$R_\mathrm{f} = K_\mathrm{k}(s) = \frac{\mathscr{K}(s)}{\mathscr{B}(s)} = \frac{\mathscr{R}(s) - \mathscr{A}(s)}{\mathscr{B}(s)} = \frac{\boldsymbol{k}^\mathrm{T}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}}{\boldsymbol{c}^\mathrm{T}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{b}} \tag{16}$$

and the calibration factor is

$$k_\mathrm{r} = \frac{\boldsymbol{k}^\mathrm{T}\boldsymbol{A}^{-1}\boldsymbol{b} - 1}{\boldsymbol{c}^\mathrm{T}\boldsymbol{A}^{-1}\boldsymbol{b}} = \frac{1 + K_\mathrm{k}(0)P(0)}{P(0)} \tag{17}$$

Given the block schemes of Fig. 2 it can be stated that the state feedback also stabilizes the unstable terms, since due to the effect of the polynomial $\mathscr{K}(s) = \mathscr{R}(s) - \mathscr{A}(s)$ there is a pole placement for any process, so with the stable $\mathscr{R}(s)$ the stabilization is fulfilled. The feedback polynomial $\mathscr{K}(s)$ formally corresponds to $\boldsymbol{k}^\mathrm{T}$. The fact that the numerator $\mathscr{B}(s)$ of the process is present in the denominator of $K_\mathrm{k}(s)$ needs special consideration. The regulator can be applied only for minimum phase (inverse stable) processes, where the roots of $\mathscr{B}(s)$ are stable. As a consequence of this special character of the state feedback, however, here $\mathscr{B}(s)$ is not substituted by its model $\hat{\mathscr{B}}(s)$, but the method itself realizes the exact $1/\mathscr{B}(s)$.

3.2. **Pole placement with pole cancellation.** The model of the system is characterized by the transfer function, which gives the relationship between the input and the output of the system.

Consider the closed control system shown in Fig. 3, where the regulator $C = \mathscr{A}/\mathscr{X}$ is used to place the poles of the closed control system according to the characteristic equation $\mathscr{R} = 0$, ($\mathscr{R}$ is the design polynomial) by the cancellation of the process poles. To do this, $\mathscr{X}$ needs to be expressed by the equation $\mathscr{R} = \mathscr{X} + \mathscr{B}$. The complementary sensitivity function of the closed-loop is

$$T = \frac{\frac{\mathscr{A}}{\mathscr{X}}\frac{\mathscr{B}}{\mathscr{A}}}{1 + \frac{\mathscr{A}}{\mathscr{X}}\frac{\mathscr{B}}{\mathscr{A}}} = \frac{\mathscr{A}\mathscr{B}}{\mathscr{A}\mathscr{X} + \mathscr{A}\mathscr{B}} = \frac{\mathscr{B}}{\mathscr{X} + \mathscr{B}} = \frac{\mathscr{B}}{\mathscr{R}}. \tag{18}$$

The regulator is

$$C = \frac{\mathscr{A}}{\mathscr{X}} = \frac{\mathscr{A}}{\mathscr{R} - \mathscr{B}} = \frac{\frac{\mathscr{B}}{\mathscr{R}}}{1 - \frac{\mathscr{B}}{\mathscr{R}}}\frac{\mathscr{A}}{\mathscr{B}} = \frac{R_\mathrm{r}}{1 - R_\mathrm{r}}P^{-1} \tag{19}$$

and actually corresponds to an ideal YOULA regulator (see later) with reference model $R_\mathrm{r} = R_\mathrm{n} = \mathscr{B}/\mathscr{R}$. This regulator places the poles in $\mathscr{R}$ and leaves the zeros in $\mathscr{B}$ untouched, if they are inverse stable.

It has to be emphasized that the poles of the system have to be stable. Only stable poles of the system model can be cancelled by the regulator.

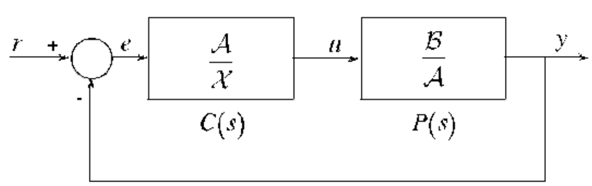

FIGURE 3. Pole cancelling regulator

With a feedback structure not only reference signal tracking, but also disturbance rejection can be ensured. Fig. 4 gives the feedback system showing also the disturbances acting on the input and the output of the system and the measurement noise as well. The reference signal can be modified by a filter.

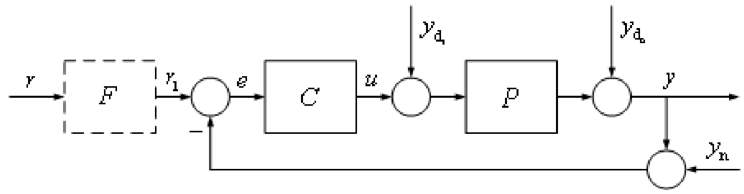

FIGURE 4. Pole cancelling regulator

A special case of pole cancellation is the use of the $PI(D)$ controllers where generally not the whole denominator of the process is cancelled. In the industry these are the most frequently used control algorithms. In continuous systems the transfer function of a *PID* controller is

$$C(s) = k_{\mathrm{c}}\left(1 + \frac{1}{sT_{\mathrm{I}}} + \frac{sT_{\mathrm{D}}}{1+sT_1}\right) \approx k_{\mathrm{c}}\frac{1+sT_{\mathrm{I}}}{sT_{\mathrm{I}}} \cdot \frac{1+s\left(T_{\mathrm{D}}+T_1\right)}{1+sT_1}; \quad T_{\mathrm{I}} > T_D > T_1. \tag{20}$$

The first, proportional (P) term of the controller reflects to the actual value of the error signal, the second, integrating (I) term utilizes the information about the past course of the error signal, while the last, differentiating (D) term reflects to the trend of the change of the error signal. The differentiating effect is not ideal, which is unrealizable, but appears together with a time lag. In a *P* controller only the first term of the algorithm is used. *PI* controller applies the proportional and the integral part, *PD* controller uses the proportional and the differentiating part, while in *PID* controller all the three terms do appear.

The control algorithm can be approximated by serially connected *PI* and phase lead terms if the given relations for the time constants do exist (20).

A usual, effective control strategy (compensation) is to cancel the "bad" (slow) poles of the process by the zeros of the controller introducing better poles (faster poles or integrator) instead. Generally not the whole denominator of the process model is cancelled. *PI* controller cancels the biggest time constant term of the process and in its denominator introduces an integrating effect. *PID* controller cancels the two biggest time constant terms of the process and introduces in its denominator an integrating effect and a smaller time constant. The gain k_c of the controller is designed to ensure stability and good phase margin (generally $\sim 60^\circ$) for the control system.

If the process does not contain integrating effect, integrating *I* effect is needed in the controller to track the step reference signal without error in steady state.

The *PID* controller modifies the poles of the system, but leaves the zeros of the process model unchanged.

3.3. **Pole placement with feedback regulator.** Another solution for pole placement is when the regulator is put in the feedback of the process as shown in Fig. 5. The task is again to place the poles of the closed system according to the equation $\mathscr{R} = 0$ ($\mathscr{R}$ is the design polynomial). To do this, $\mathscr{K}$ needs to be determined from the equation $\mathscr{R} = \mathscr{K} + \mathscr{A}$. The complementary sensitivity function of the closed system is

$$T = \frac{\frac{\mathscr{B}}{\mathscr{A}}}{1 + \frac{\mathscr{K}}{\mathscr{B}}\frac{\mathscr{B}}{\mathscr{A}}} = \frac{\mathscr{B}}{\mathscr{A}+\mathscr{K}} = \frac{\mathscr{B}}{\mathscr{R}} \tag{21}$$

and thus this regulator places the poles in $\mathscr{R}$ and leaves the zeros in $\mathscr{B}$ untouched, if they are inverse stable.

The characteristic equation of the closed system has the form $\mathscr{R} = 0$ and it does not depend on the unstable property of the process.

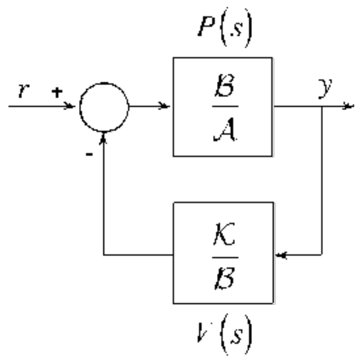

FIGURE 5. Regulator in the feedback

The block diagram in Fig. 5 can be redrawn according to Fig. 6, which is equivalent to Fig. 2(c). (The state feedback methods are discussed in detail in Section 3.1, and the same control principle is represented in Fig. 2(c) among the schemes showing the equivalent transfer function representations for state feedback.)

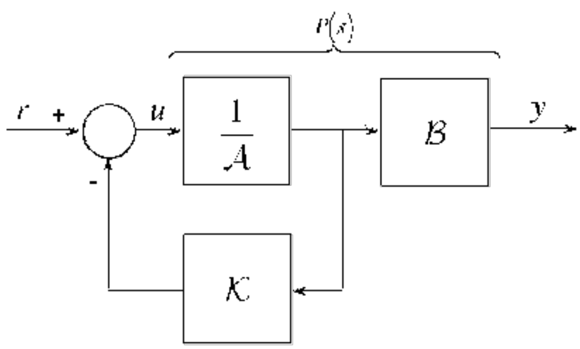

FIGURE 6. The regulator feeds back the internal signal of the process

3.4. **Pole placement with characteristic polynomial design.** The characteristic polynomial $\mathscr{R}$ of the closed-loop control can be directly designed by algebraic methods. In Fig. 7 the regulator $C = \mathscr{Y}/\mathscr{X}$ is the quotient of two polynomials. Under certain conditions, the *DE* (Diophantine Equation) $\mathscr{A}\mathscr{X} + \mathscr{B}\mathscr{Y} = \mathscr{R}$ can be solved for $\mathscr{X}$ and $\mathscr{Y}$. Thus from the characteristic equation $\mathscr{R} = 0$ the regulator can be directly determined.

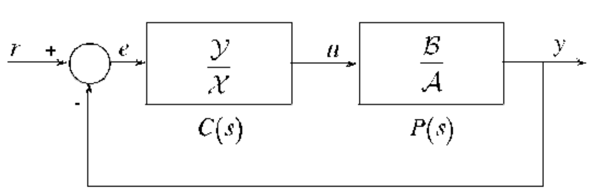

FIGURE 7. Direct control design on the basis of the characteristic polynomial

The complementary sensitivity function of the closed system is

$$T = \frac{\mathscr{Y}\mathscr{X}\frac{\mathscr{B}}{\mathscr{A}}}{1+\frac{\mathscr{Y}}{\mathscr{X}}\frac{\mathscr{B}}{\mathscr{A}}} = \frac{\mathscr{B}\mathscr{Y}}{\mathscr{A}\mathscr{X}+\mathscr{B}\mathscr{Y}} = \frac{\mathscr{B}\mathscr{Y}}{\mathscr{R}} \tag{22}$$

and thus this regulator also places the poles in $\mathscr{R}$ and leaves the zeros in $\mathscr{B}$ untouched, but in the nominator $\mathscr{Y}$ appears, which depends on the desired properties and also on the DE.

Thus the characteristic equation of the closed system has the form $\mathscr{R} = 0$ and it does not depend on the unstable character of the process.

4. REGULATORS BASED ON YOULA PARAMETERIZATION

The YOULA parameter is a stable, regular transfer function defined as (11) and (12)

$$Q(s) = \frac{C(s)}{1+C(s)P(s)} \quad \text{or shortly} \quad Q = \frac{C}{1+CP}, \tag{23}$$

where $C(s)$ is a stabilizing regulator, and $P(s)$ is the transfer function of the stable process (see Figs 3,4). In a simple feedback system the YOULA parameter gives the relationship between the control signal and the reference signal.

It follows from the definition of the YOULA parameter that the structure of the realizable and stabilizing regulator in the YOULA-parameterized (sometimes called Q-parameterized) control loop is

$$C(s) = \frac{Q(s)}{1-Q(s)P(s)} \quad \text{or shortly} \quad C = \frac{Q}{1-QP}. \tag{24}$$

It is interesting to observe that the YP regulator of (24) can be realized by a simple control loop with positive feedback as shown in Fig. 8.

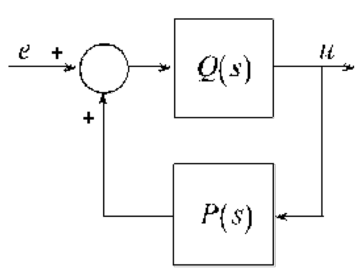

FIGURE 8. Realization of a YP regulator

A YOULA-parameterized (YP) closed-loop is shown in Fig. 9.

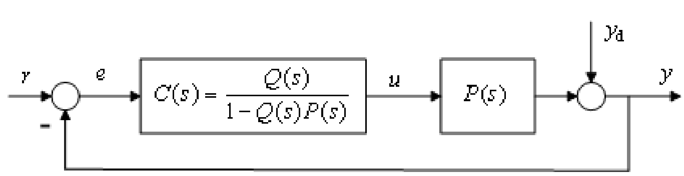

FIGURE 9. YOULA-parameterized closed-loop

The *A̲ll-R̲ealizable-S̲tabilizing* (*ARS*) regulator has the form of (24). The closed-loop transfer function or *C̲omplementary S̲ensitivity F̲unction* (*CFS*) is

$$T = \frac{CP}{1+CP} = QP. \tag{25}$$

The sensitivity function is

$$S = \frac{1}{1+CP} = 1 - T = 1 - QP \tag{26}$$

which are linear in the YOULA parameter Q.

The YP regulator determines the reference signal tracking according to (25), so Q can be designed in open loop. Disturbance rejection is ensured in the classical *IMC* (Internal Model Control, [2]) structure, according to Fig. 10, which is equivalent to Fig. 9.

The relationships between the most important signals of the closed system can be obtained with simple calculations.

$$\begin{aligned} u &= Qr - Qy_{\mathrm{d}} \\ e &= (1-QP)r - (1-QP)y_{\mathrm{d}} = Sr - Sy_{\mathrm{d}} \\ y &= QPr + (1-QP)y_{\mathrm{d}} = Tr + Sy_{\mathrm{d}} \end{aligned} \tag{27}$$

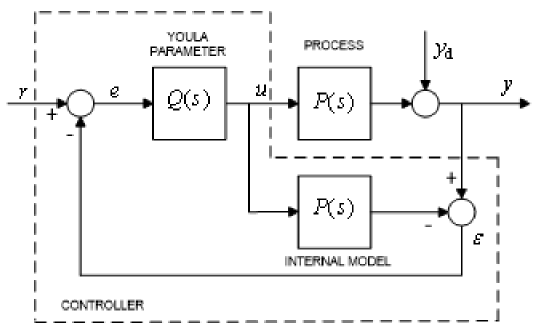

FIGURE 10. *IMC* form of YOULA parameterization

The effect of r and y_{d} on u and e is completely symmetrical (not considering the sign). Thus the input of the process depends only on the external signals and on $Q(s)$.

Ideal control performance could be reached if the YOULA parameter would be the inverse of the transfer function of the process model. But generally this inverse is non-realizable. The transfer function of the process should be separated to the invertible P_+ and the non-invertible $\bar{P}_-$ part which contains the inverse unstable zeros and the dead time also (the upper line indicates the existence of dead time). The inverse is also non-realizable if its numerator is of higher degree than the degree of its denominator. The YOULA parameter can be chosen as the inverse of the invertible part of the process.

It is mentioned that *PID* control can be considered as a special case of YOULA parameterization, where C_{PID} is the series controller in the feedback control loop, and the YOULA parameter is $Q = \frac{C_{\mathrm{PID}}}{1+C_{\mathrm{PID}}P}$. If $C_{\mathrm{PID}}P \gg 1$ (which is fulfilled generally in the low frequency domain), then $Q \approx P^{-1}$.

Reference signal filter R_{r} and disturbance filter R_{n} can be introduced to make different transfer properties for reference signal tracking and disturbance rejection (Figs. 11, 12). The static gain of the filters should be 1 . In this case the control system is called 2DOF - two dimension of freedom control system. R_r determines the behavior for reference signal tracking, while R_n shapes the transient for disturbance rejection.

Another role of the filters is to modify the value of the control signal u keeping it below the allowed maximum value. Also the filters have a robustifying effect. With their appropriate choice the control system can be done less sensitive to plant/model mismatch.

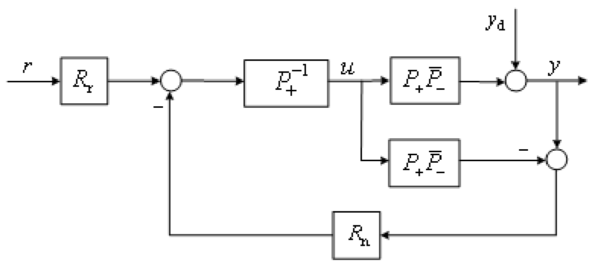

FIGURE 11. *IMC* form of YOULA parameterization with reference and disturbance filters

Equivalent form of the block diagram is given in Fig. 12 .
Now the YOULA parameter is $Q = R_n P_+^{-1}$.

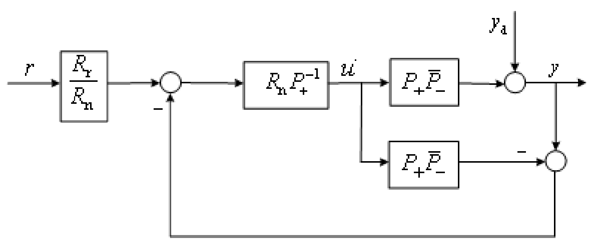

FIGURE 12. Equivalent *IMC* form of Youla parameterization with filters

Keviczky and Bányász introduced a modified form of YOULA parameterization, called KB parameterization [7] according to Fig. 13. This structure "opens the loop" between signals y and r', the transfer function between these points is P. Then a parameter Q_r is applied for the design of the tracking properties, connecting it in serial to the KB-parameterized loop. Disturbance rejection properties are influenced by parameter Q. This control structure ensures two-degree-of freedom performance.

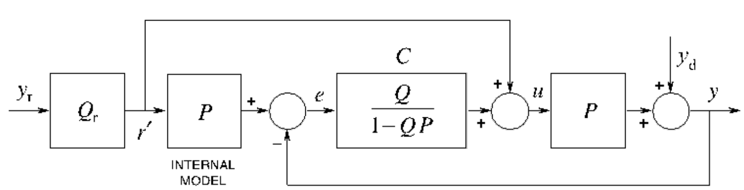

FIGURE 13. KB-parameterized control loop

The overall transfer characteristics for this system are

$$\begin{aligned} u &= Q_{\mathrm{r}} y_{\mathrm{r}} - Q y_{\mathrm{d}}, \\ e &= (1 - Q_{\mathrm{r}} P)\, y_{\mathrm{r}} - (1 - QP) y_{\mathrm{d}} = (1 - T_{\mathrm{r}})\, y_{\mathrm{r}} - S y_{\mathrm{d}}, \\ y &= Q_{\mathrm{r}} P y_{\mathrm{r}} + (1 - QP) y_{\mathrm{d}} = T_{\mathrm{r}} y_{\mathrm{r}} + (1 - T) y_{\mathrm{d}} = T_{\mathrm{r}} y_{\mathrm{r}} + S y_{\mathrm{d}}, \end{aligned} \tag{28}$$

where the tracking properties can be designed by choosing Q_{r} in $T_{\mathrm{r}} = Q_{\mathrm{r}} P$, and the noise rejection properties by choosing Q in $T = QP$. These two properties can be handled separately. The reference signal of the whole system is denoted by y_{r}. The conditions for Q_{r} are the same as for Q. The meaning of T_{r} is analogous to the meaning of the complementary sensitivity function T of the one-degree-of-freedom control loop for tracking ([5], [6], and [7]).

4.1. **Computation of the optimal YOULA regulator.** Let us analyze the design of the YOULA regulator in more detail for LTI systems.

The transfer function of the process can be written in the following factorized form

$$P(s) = P_{+}(s)\bar{P}(s)_{-} = P_{+}(s)P_{-}(s)e^{-sT_{\mathrm{d}}}, \tag{29}$$

where T_{d} denotes the dead time. Shortly,

$$P = P_{+}\bar{P}_{-} = P_{+}P_{-}e^{-sT_{\mathrm{d}}}, \tag{30}$$

where P_{+} is stable, and its inverse is also stable (*Inverse Stable*: *IS*) and realizable (*ISR*). The inverse of $\bar{P}_{-}$ is unstable (*Inverse Unstable*: *IU*) and non-realizable (*Non Realizable*: *NR*), i.e., (*IUNR*). P_{-} is inverse unstable (*IU*). The inverse of the dead-time part $e^{-sT_{\mathrm{d}}}$ is non-realizable, because it would be an ideal predictor.

In polynomial form a delay free process is given by

$$P(s) = \frac{\mathscr{B}(s)}{\mathscr{A}(s)} = \frac{\mathscr{B}_{+}(s)\mathscr{B}_{-}(s)}{\mathscr{A}(s)}, \tag{31}$$

where $\mathscr{B}_{+}(s)$ and $\mathscr{B}_{-}(s)$ contain the inverse stable and inverse unstable zeros, respectively. Here $P_{+}(s) = \frac{\mathscr{B}_{+}(s)}{\mathscr{A}(s)}$ and $P_{-}(s) = \mathscr{B}_{-}(s)$. The reference model, formulating our design goal is

$$R_n(s) = \frac{\mathscr{B}_n(s)}{\mathscr{A}_n(s)}. \tag{32}$$

The optimal YOULA parameter is

$$Q(s) = R_{\mathrm{n}}(s)P_{+}^{-1} = R_{\mathrm{n}}(s)\mathscr{B}_{+}^{-1}(s)\mathscr{A}(s). \tag{33}$$

Using this parameterization, the optimal YOULA regulator can be calculated as

$$C(s) = \frac{Q(s)}{1 - Q(s)P(s)} = \frac{R_{\mathrm{n}}(s)\mathscr{B}_{+}^{-1}(s)\mathscr{A}(s)}{1 - R_{\mathrm{n}}(s)\mathscr{B}_{+}^{-1}(s)\mathscr{B}_{+}(s)\mathscr{B}_{-}(s)} = \frac{\mathscr{B}_{\mathrm{n}}(s)\mathscr{A}(s)}{\mathscr{B}_{+}(s)\left[\mathscr{A}_{\mathrm{n}}(s) - \mathscr{B}_{\mathrm{n}}(s)\mathscr{B}_{-}(s)\right]}. \tag{34}$$

The transfer function of the closed-loop system in the 2DF system (Fig. 12)

$$T(s) = R_{\mathrm{r}}(s)\mathscr{B}_{-}(s) = \frac{\mathscr{B}_{\mathrm{r}}(s)}{\mathscr{A}_{\mathrm{r}}(s)}\mathscr{B}_{-}(s), \tag{35}$$

which is the best reachable result for the case of inverse unstable zeros. This result explains the name: ”non-cancellable” for the inverse unstable factors of the numerator of the process.

It can be seen that the computation of the YOULA regulator requires only very simple polynomial operations (additions and multiplications).

5. ABOUT DISCRETE CONTROL ALGORITHMS

In practice frequently the control systems are realized by digital computers equipped with appropriate real-time facilities. The main functions of a control system are executed by the digital computer in real time.

The output of the system is sampled and connected to the computer via A/D converter. The computer executes in every sampling time the tasks of creating the reference signal, accepting and maybe also filtering the measured signal, calculating the difference between the reference signal and the measured output signal, and based on this error signal calculates the actuating signal according to a control algorithm which is realized by a program. Then it provides this signal toward the actuator of the process via a D/A converter. The process is continuous, thus generally a zero order hold keeps the actual control signal until the next value does appear in the next sampling point. The block diagram of the sampled (DT – discrete-time) system is given in Fig. 14.

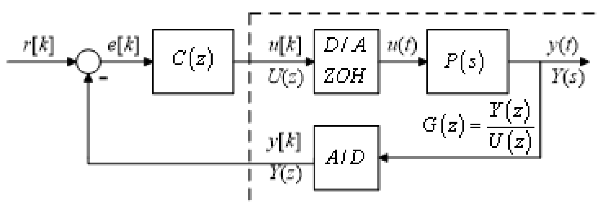

FIGURE 14. Block diagram of sampled system (computer control)

The requirements set for the control system are the same as in the case of continuous control. The difference is that because of sampling, quantization and digitalization there is a loss in the information which is available only in the sampling times. It is important to choose the sampling time appropriately, according to the Shannon theorem. The choice of the sampling time influences very important properties of the control loop as stability, settling time, maximum value of the control signal. Sampling behaves as if an extra dead time has been introduced in the control system, which is about the half of the sampling time.

The model of the system can be the discrete state equation or in input/output representation the pulse transfer function which is the ratio of the z-transforms of the output and of the input signal respectively. Similar control algorithms can be considered as for the continuous case but using the discrete models (e.g. the pulse transfer function instead of the transfer function) [1].

In the industrial applications the mostly used control algorithms are the *PID algorithms*. In discrete control systems they can be realized by discretizing the continuous algorithms or by using the pole cancellation technique for the discrete model similarly to compensation of continuous systems. The *YOULA parameterized control algorithms* can be calculated with the pulse transfer functions. The YOULA parameterization provides a straightforward analytical controller design algorithm. For discrete systems zeros of the pulse transfer function of the process which lie outside of the unit circle cannot be inverted, and cancellation of zeros which lie in the left side of the unit circle is to be avoided as their inversion would cause intersampling oscillation.

6. IMPROVING DISTURBANCE REJECTION

The series control structure based on negative feedback shown in Fig. 4 ensures reference signal tracking and disturbance rejection as well. If along the path from the disturbance to the output signal components with big time constants do exist, then the disturbance rejection will be slow. The effect of the disturbances has to appear in the output signal in order to start actuating against it. Disturbance rejection can be improved if not only the effects caused by the disturbance in the output signal are utilized for disturbance rejection, but possibly some outer or inner measurable signals are also used in which the effect of the disturbance appears already earlier than in the output signal. Utilizing available information in the control circuit the quality of the control system can be improved.

6.1. **Feedforward.** If the disturbance is measurable, with feedforward the quality of the control system, especially its disturbance rejection properties, can be significantly improved. Based on the measured value of the disturbance it is possible to execute actions to reject it before its effect would appear in the controlled variable. The block diagram of feedforward control is shown in Fig. 15. Here the closed-loop system is extended by an open-loop path.

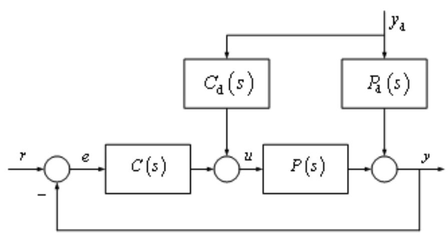

FIGURE 15. Block diagram of feedforward control

The disturbance acts on the output through two paths. With appropriate design of the feedforward controller $C_d(s)$ the effect of the disturbance can significantly be decreased or even totally compensated. The effect of the disturbance will not appear in the output signal if the following relationship is fulfilled:

$$P_d(s) + C_d(s)P(s) = 0 \text{ hence } C_d(s) = -\frac{P_d(s)}{P(s)}. \tag{36}$$

If this transfer function is realizable (the degree of its numerator is not higher than the degree of its denominator, furthermore $P(s)$ does not contain dead time), the effect of the disturbance does not appear at all in the output signal. If $C_d(s)$ is non-realizable, its transfer function has to be approximated by the best realizable controller. Feedforward control is widely used in the industry.

6.2. **Cascade control.** Several times the processes can be separated to serially connected parts, and besides the output signal the intermediate signals can also be measured. Fig. 16 shows the block diagram of a process which consists of two serially connected parts. The disturbances may act on the output or between the two parts of the process. It is supposed that the disturbances themselves are not measurable.

It is worthwhile to use the additional information about the inner process signal to create an inner control loop. Fig. 17 shows this structure called cascade control.

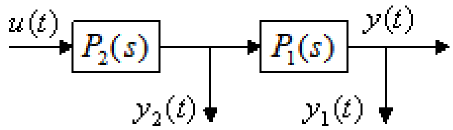

FIGURE 16. A process which can be separated to two serially connected parts

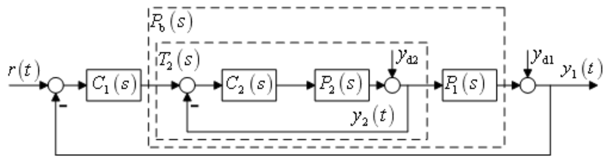

FIGURE 17. A process which can be separated to two serially connected parts

As the effect of the inner disturbance appears sooner in signal y_2 than in the output y_1, the inner loop can rather quickly decrease its effect. The outer loop ensures good reference signal tracking, the rejection of the output disturbance and further attenuation of the effect of the inner disturbance which has been already decreased by the inner loop. The advantage of cascade control compared to a single-loop feedback control is manifested if part $P_1(s)$ of the plant contains the big time constants and/or dead time, while part $P_2(s)$ contains the smaller time constants. $C_2(s)$, the controller of the inner loop is designed for fast performance of the inner loop, thus the inner loop will quickly reject the inner disturbance. With controller $C_1(s)$ of the outer loop good reference signal tracking and rejection of the outer disturbance is to be ensured. The inner controller could be of structure *P* or *PD*. The controller in the outer loop which ensures the quality specifications could be of structure *PI* or *PID*. Also YOULA parameterized controllers can be used. In some applications it is expedient to put saturation after the outer controller. As the output of the outer controller provides the reference signal of the inner loop, restricting its value the inner signal y_2 can also be kept within prescribed limits.

Of course if the process can be separated to more than two components, where the inner signals are measurable, cascade control can be realized with several nested control circuits.

7. COMPARISON OF THE PREVIOUSLY DISCUSSED DESIGN METHODS

Control loops with state feedback.

The most important advantage of the state feedback regulator is that the calculation of the feedback vector is very simple. The most important disadvantage is that the internal state variables, necessary for the feedback are usually not available in the practical tasks. This is why the observer topology is generally necessary to estimate the states. Unfortunately it is not so simple to compute this topology. This regulator assigns the poles of the closed-loop system and leaves the numerator of the process untouched in T. It is important to know that from the methods discussed in this paper this is the only method which is applicable for unstable processes.

Pole placement with pole cancellation.

The most important advantage of this method is that it is very simple to calculate the regulator. The disadvantage is that this regulator assigns the poles of the closed-loop system, unfortunately it also leaves the numerator of the process untouched in T. *PID* control is a well known version of this method.

Pole placement with feedback regulator.

This method practically can be evaluated in similar way as the previous method. The most important disadvantage is that in a practical task it is very rare that the regulator is in the feedback line.

Pole placement with characteristic polynomial design.

This method is a little bit more complex than the pole cancellation method, because the calculation of the regulator needs the solution of a DE. The disadvantage is that this regulator assigns the poles of the closed-loop system, unfortunately it also leaves the numerator of the process untouched in T and puts another polynomial in the numerator of T. This polynomial comes from the solution of the DE, so it is not easy to design.

Regulators based on YOULA parameterization.

This method is the simplest, because it needs only basic polynomial operations to calculate the regulator. A further advantage is that the result of the design is the best reachable T even for invariant process zeros.

Except the state feedback regulator the other methods are applicable only for stable processes.

It is mentioned, that with the extension of the YOULA parameterization using general polynomial methods the YOULA-KUCERA parameterization can be applied where solving a DE equation control of unstable plants can also be realized [13].

8. SIMULATION EXAMPLES

In the following several examples illustrate the controller design methods.

The calculations can be supported by commands of the MATLAB control toolbox. The SIMULINK program included in MATLAB software is used for simulations.

Example 8.1. The CT process is given by the transfer function

$$P(s) = \frac{6}{(s+1)(s+2)(s+3)}. \tag{37}$$

Let us design regulators with the discussed methods and compare their behavior.

a./ Pole placement with state feedback.

The step response of the plant is shown in Fig. 18. By state feedback the behavior can be accelerated if the poles of the closed loop system are shifted to the left on the complex plane. Let the prescribed poles of the closed loop system be: $-6; -3+4i; -3-4i$. The complex conjugate poles provide a damping factor of 0.6 ensuring a small overshoot in the step response. Thus the prescribed characteristic polynomial is

$$\mathscr{R}(s) = s^3 + 12s^2 + 61s + 150 \tag{38}$$

The controllable canonical form of the state equation is:

$$\boldsymbol{A} = \begin{bmatrix} -6 & -11 & -6 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}; \quad \boldsymbol{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{39}$$
$$\boldsymbol{c} = \begin{bmatrix} 0 & 0 & 6 \end{bmatrix}; d = 0.$$

The state feedback vector calculated according to (11) is

$$\boldsymbol{k} = \begin{bmatrix} 6 & 50 & 144 \end{bmatrix}^{\mathrm{T}} \tag{40}$$

and the value of the calibration factor calculated according to (8) is 25 .

The step response of the plant and of the controlled system is shown in the left side of Fig. 18. It is seen that with the control the settling process became faster. The control signal is given on the right side of the figure. The overexcitation in the control signal ensures the acceleration.

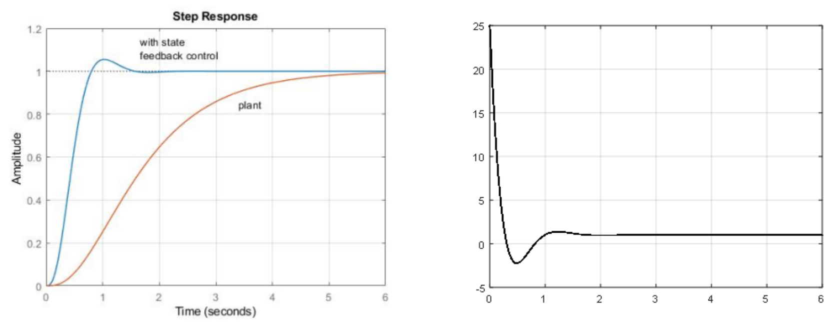

FIGURE 18. Step response and the control signal with state feedback control

b./ Pole placement with pole cancellation.

According to (19)

$$C = \frac{\mathscr{A}}{\mathscr{X}} = \frac{\mathscr{A}}{\mathscr{R}-\mathscr{B}} = \frac{\frac{\mathscr{B}}{\mathscr{R}}}{1-\frac{\mathscr{B}}{\mathscr{R}}}\frac{\mathscr{A}}{\mathscr{B}} = \frac{R_{\mathrm{r}}}{1-R_{\mathrm{r}}}P^{-1} \tag{41}$$

In order to eliminate the steady state error let us divide the characteristic polynomial R by a factor which ensures that its constant term will be equal to $\mathrm{B}(0)$, so the controller will be of integral type.

$$R_{\mathrm{m}}(s) = \left(s^3 + 12s^2 + 61s + 150\right)/25. \tag{42}$$

So the transfer function of the controller is

$$C(s) = \frac{s^3 + 6s^2 + 11s + 6}{0.04s^3 + 0.48s^2 + 2.44s}. \tag{43}$$

The step response is shown in Fig. 19 and the control signal is given in Fig. 20. It is seen that overexcitation in the control signal ensures acceleration of the output signal. Of course there is a practical limit of the control signal provided by the actuator.

It is seen that the performance is similar in the two cases. In case b./ the output signal is used, while in case a./ the state variables should be reached. Similar result can be obtained with the feedback regulator.

c./ Pole placement with PID regulator.

According to pole cancellation technique the controller cancels the biggest time constant of the process and introduces an integrating effect instead and the second biggest time constant is also cancelled and substituted with a smaller time constant. The gain of the controller is chosen to ensure approximately 60° phase margin. So the transfer function of the controller is

$$C(s) = \frac{1+s}{s} \cdot \frac{1+0.5s}{1+0.333s} \tag{44}$$

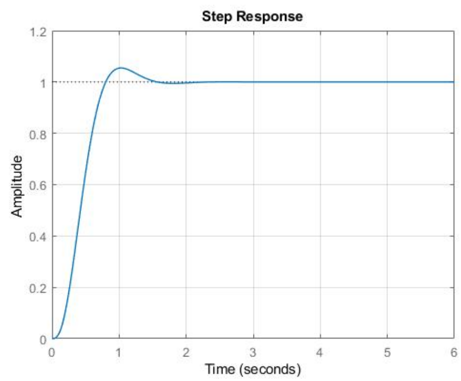

FIGURE 19. Step response with pole cancellation

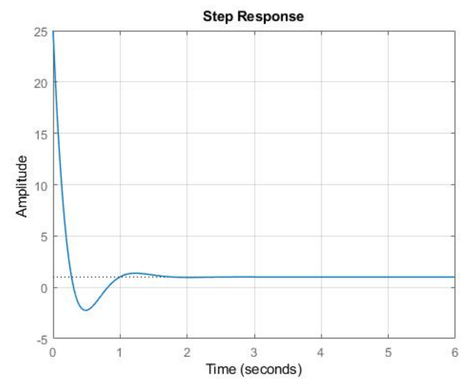

FIGURE 20. Control signal

The output and the control signal are shown in Fig. 21.

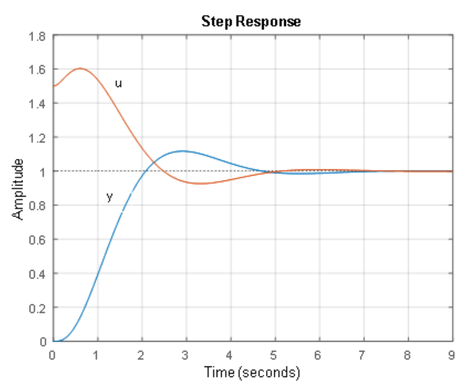

FIGURE 21. Output and control signal with *PID* compensation

It is seen that the settling process is slower than with the previous methods.

d. / YOULA parameterized regulator design.

Now the whole transfer function is invertible. It is worthwhile to choose third order filters to get realizable YOULA parameter.

Let the transfer function of the reference signal filter be

$$R_{\mathrm{r}}(s) = \frac{150}{s^3 + 12s^2 + 61s + 150}, \tag{45}$$

which ensures the poles of the characteristic equation as shown in point *a.*/

Let the transfer function of the noise filter be

$$R_{\mathrm{n}}(s) = \frac{64}{(s+4)^3} = \frac{64}{s^3 + 12s^2 + 48s + 64}. \tag{46}$$

The YOULA parameter is

$$Q(s) = R_{\mathrm{n}}(s)P^{-1}(s) = \frac{64}{6}\frac{(s+1)(s+2)(s+3)}{s^3 + 12s^2 + 48s + 64}. \tag{47}$$

The output signal as response to a unit step reference signal and the output signal as a response to unit step disturbance is shown in Fig. 22.

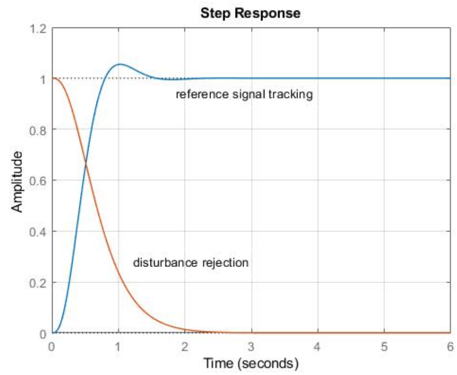

FIGURE 22. Unit step reference signal tracking and disturbance rejection of the YOULA parameterized controller

The performance is similar to that obtained by state feedback, but the design procedure is much simpler. The controller transfer function is obtained in a straightforward way by polynomial operations in the knowledge of the plant transfer function and the filters. In this case $T(s) = R_{\mathrm{r}}(s)$.

Example 8.2. Let us analyze the control of a first order process with dead time applying *PI* control and YOULA parameterized control algorithm both for CT and DT time cases.

The CT process is given by the transfer function

$$P(s) = \frac{1}{1+T_1 s} e^{-T_{\mathrm{D}} s} = \frac{1}{1+5s} e^{-30s} \tag{48}$$

Aperiodic processes frequently are approximated with this model.

a./ Let us design continuous controllers.

With *PI* controller pole cancellation technique can be applied. The time constant term of the process is cancelled and an integrating effect is introduced instead. The gain of the controller is chosen to ensure approximately phase margin of 60°. This is reached if the cut-off frequency in the Bode diagram of the open loop is approximately $\omega_c = 1/2\,\mathrm{T_D} = 1/60$.

$$C(s) = k_{\mathrm{c}} \frac{1+5s}{5s}. \tag{49}$$

The loop transfer function is

$$L(s) = C(s)P(s) = k_{\mathrm{c}} \frac{e^{-30s}}{5s}. \tag{50}$$

According to the design $k_{\mathrm{c}}/5\omega_{\mathrm{c}} = 1$, whence $k_{\mathrm{c}} = 5/60 = 0.083$. The settling time is between $3/\omega_{\mathrm{c}}$ and $10/\omega_{\mathrm{c}}$: $180 < t_{\mathrm{s}} < 600$sec.

The simulation can be executed with MATLAB/SIMULINK. The output and the control signals are shown in Figs 23 and 24. The dead time limits the speed of the settling process.

Let us design now a YOULA parameterized controller. The dead time is the non-invertible part of the process. So

$$P_{+}(s) = \frac{1}{1+5s} \text{ and } P_{-}(s) = e^{-30s}. \tag{51}$$

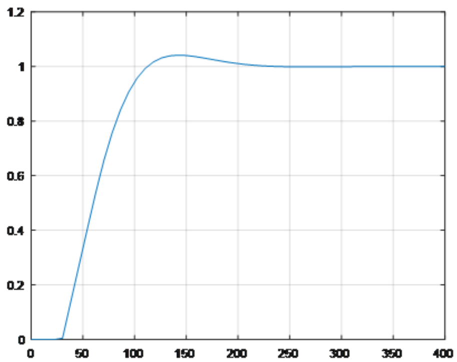

FIGURE 23. The output signal, *PI* control

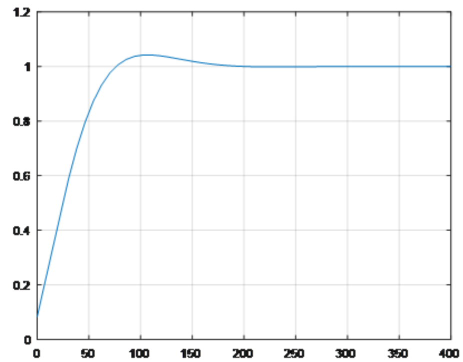

FIGURE 24. The control signal, *PI* control

Let us choose the reference signal filter as $R_{\mathrm{r}}(s)=\frac{1}{1+2s}$ and the disturbance filter as $R_{\mathrm{n}}(s)=\frac{1}{1+s}$. These filters ensure faster dynamics both for reference signal tracking and disturbance rejection than the dynamics of the process.

The YOULA parameter is:

$$Q=R_{\mathrm{n}}P_{+}^{-1}=\frac{1}{1+s}(1+5s) \tag{52}$$

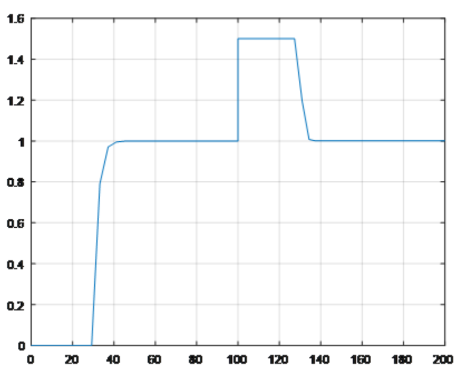

FIGURE 25. Output signal, YOULA controller

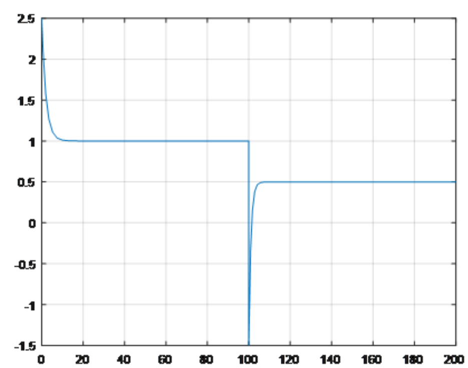

FIGURE 26. Control signal, YOULA controller

A SIMULINK diagram is built to simulate the control system. A unit step reference signal is applied and then in time point 100sec a step disturbance of 0.5 amplitude is acting. The output signal is shown in Fig. 25 and the control signal is given in Fig. 26. It is seen that the settling process is much faster than with the *PI* controller.

b./ Let us design discrete controller for the same first order process with dead time.

The sampling time is $T_{\mathrm{s}}=1$sec. The pulse transfer function of the process is

$$G(z)=\frac{0.1813}{z-0.8087}z^{-30} \tag{53}$$

The pulse transfer function of the discrete *PI* controller using pole cancellation is

$$C(z)=k_{\mathrm{c}}\frac{z-0.8087}{z-1}. \tag{54}$$

The loop transfer function is

$$L(z) = C(z)G(z) = \frac{k_c \cdot 0.1813}{z-1} \cdot z^{-30}. \tag{55}$$

The gain factor k_c of the controller can be chosen based on the following considerations in the frequency domain. The frequency function of the integrator is:

$$\frac{T_s}{z-1}\frac{1}{j\omega}e^{-j\omega T_s/2}. \tag{56}$$

That is sampling introduces a delay whose value is about half of the sampling time. The loop frequency function can be approximated by the following relationship:

$$L(j\omega) = \frac{k_c \cdot 0.1813}{j\omega T_s} \cdot e^{-30.5 j\omega}. \tag{57}$$

For phase margin of about 60° the cut-off frequency should be located at about the half of the reciprocal of the dead time. At the cut-off frequency

$$\frac{k_c \cdot 0.1813}{\omega_c T_s} = 1, \text{ whence } k_c = \frac{\omega_c T_s}{0.0813} = \frac{1}{61 \cdot 0.1813} = 0.0904 \tag{58}$$

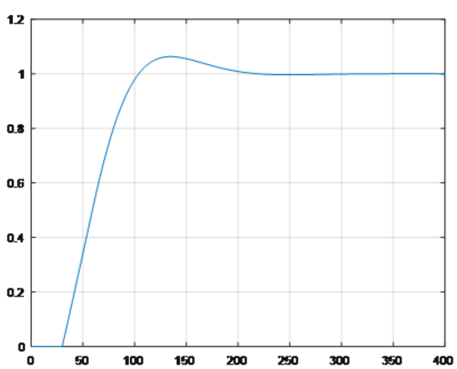

FIGURE 27. The output signal.
Discrete *PI* controller

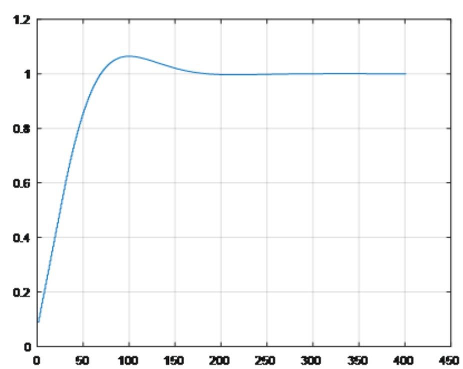

FIGURE 28. The control signal.
Discrete *PI* controller

The simulation can be executed with MATLAB/SIMULINK. The output and the control signals obtained running the SIMULINK diagram are shown in Figs 27 and 28 respectively. The process is continuous, the controller is discrete. With bigger sampling time the stepwise character of the control signal would be better seen.

Let us design now a YOULA parameterized controller with the discretised filters used for the continuous control. The pulse transfer functions of the reference filter and the disturbance filter are

$$R_r(z) = \frac{0.3935}{z-0.6065} \text{ and } R_n(z) = \frac{0.6321}{z-0.3679}. \tag{59}$$

The YOULA parameter is

$$Q(z) = R_n(z)G_+^{-1} = \frac{0.6321}{z-0.3679} \cdot \frac{z-0.8087}{0.1813} = 3.4865\frac{z-0.8087}{z-0.3679}. \tag{60}$$

To simulate the behaviour SIMULINK diagram has been built according to block diagram given in Fig. 12. The process is continuous, the controller and the filters are discrete. Between the controller and the process zero order hold is applied. A unit step reference signal is used and then in time point 100sec a step disturbance of 0.5 amplitude is acting. The output signal is given in Fig. 29 and the control signal is presented in Fig. 30.

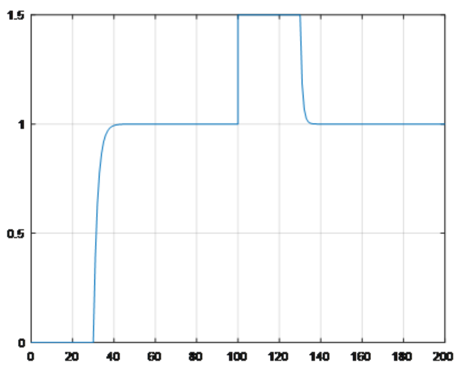

FIGURE 29. Output signal

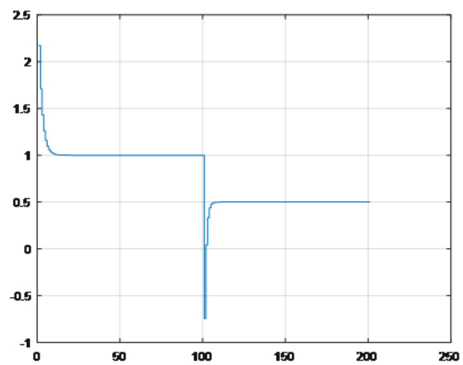

FIGURE 30. Control signal

It is seen that both in the continuous and the discrete case the YOULA parameterized controller gives much faster settling than the *PI* control. The YOULA controller design is simple.

Example 8.3. A second order continuous process with significant dead time is controlled with discrete *PID* and with YOULA controller. Let us design the controllers and analyze and compare the behavior of the control system for reference signal tracking and disturbance rejection. In case of the YOULA controller show the behavior of the control system for different filters also in case of mismatch in the dead time of the real process and its model.

The transfer function of the continuous plant is

$$P(s) = \frac{1}{(1+5s)(1+10s)} e^{-30s}. \qquad (61)$$

It is sampled, at the input zero order hold is applied. The sampling time is $T_s = 5$sec. The corresponding pulse transfer function is

$$G(z) = \frac{0.1548(z+0.6065)}{(z-0.3679)(z-0.6065)} z^{-6}. \qquad (62)$$

First a *PID* controller is designed for pole cancellation (cancelling the biggest pole of the system and introducing an integrating effect instead, and cancelling also the second pole introducing a differentiation instead) and for phase margin about $\varphi_m \approx 60^\circ$. The pulse transfer function of the controller is

$$C(z) = 0.3074 \frac{z-0.6065}{z-1} \frac{z-0.3679}{z}. \qquad (63)$$

Then let us design a YOULA controller first without filters, $R_r = R_n = 1$. Let us separate the pulse transfer function of the plant to non-invertible and invertible parts.

$$\bar{G}_-\left(z^{-1}\right) = \frac{\left(1+0.6065z^{-1}\right) z^{-1}}{1.6065} z^{-6} \quad ; \quad G_+\left(z^{-1}\right) = \frac{0.1548 \cdot 1.6065}{\left(1-0.3679z^{-1}\right)\left(1-0.6065z^{-1}\right)} \qquad (64)$$

and

$$Q\left(z^{-1}\right)=\frac{\left(1-0.3679z^{-1}\right)\left(1-0.6065z^{-1}\right)}{0.1548\cdot 1.6065}=4.0211\left(1-0.3679z^{-1}\right)\left(1-0.6065z^{-1}\right). \tag{65}$$

The filters are obtained by sampling the first order systems given by transfer functions

$$R_{\mathrm{r}}(s)=\frac{1}{1+6s} \quad \text{and} \quad R_{\mathrm{n}}(s)=\frac{1}{1+5s}. \tag{66}$$

These filters ensure a bit faster performance than the transient of the process and also a somewhat different dynamics for reference signal tracking and disturbance rejection. The corresponding pulse transfer functions of the filters are

$$R_{\mathrm{r}}\left(z^{-1}\right)=\frac{0.5654z^{-1}}{1-0.4346z^{-1}} \quad \text{and} \quad R_{\mathrm{n}}\left(z^{-1}\right)=\frac{0.6321z^{-1}}{1-0.3679z^{-1}}. \tag{67}$$

The YOULA parameter is

$$Q\left(z^{-1}\right)=\frac{0.6321\left(1-0.3679z^{-1}\right)\left(1-0.6065z^{-1}\right)z^{-1}}{\left(1-0.3679z^{-1}\right)0.1548\cdot 1.6065}=2.5418\left(1-0.6065z^{-1}\right)z^{-1}. \tag{68}$$

Fig. 31 shows the output with *PID* controller and Fig. 32 gives the outputs with the YOULA controller without and with the filters. The step reference signal acts at $t=0$sec, and a step disturbance of 0.5 amplitude acts at $t=300$sec. It is seen that the YOULA control is much faster than the *PID* control. The stepwise output signal is also higher in case of the YOULA control (its max value is 4.02 without filters and 2.54 with the filters, and it is around 1.1 for *PID* control).

Let us consider the control behavior in case of plant/model mismatch. The dead time of the system is 35sec, while in the model 30sec is considered and the controller has been designed based on this model. The *PID* controller still tolerates this inaccuracy (Fig.33), but without the filters the YOULA controller becomes quite oscillating (Fig. 34). With the given filters its behaviour is moderated and still acceptable (Fig. 35). Filters can be designed for robust performance.

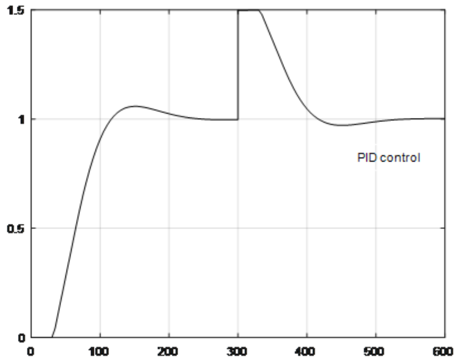

FIGURE 31. The output with *PID* control

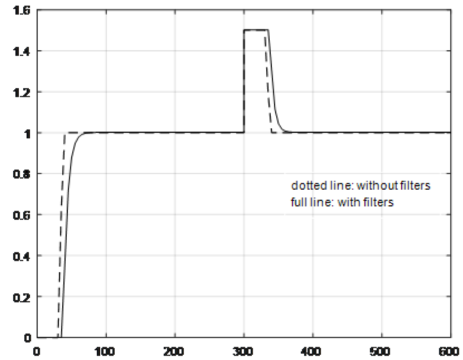

FIGURE 32. The outputs with YOULA control

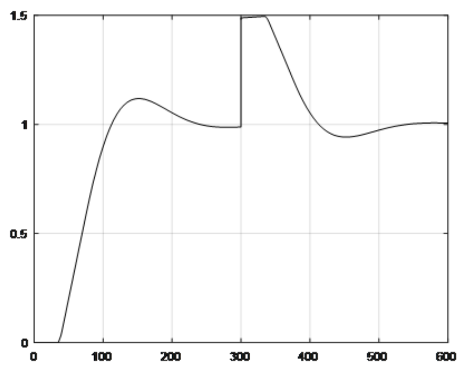

FIGURE 33. *PID* control with dead time mismatch

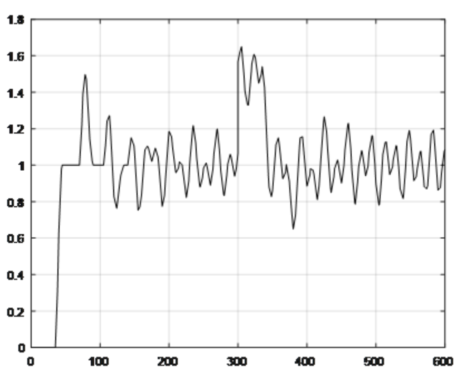

FIGURE 34. YOULA control without filters with mismatch

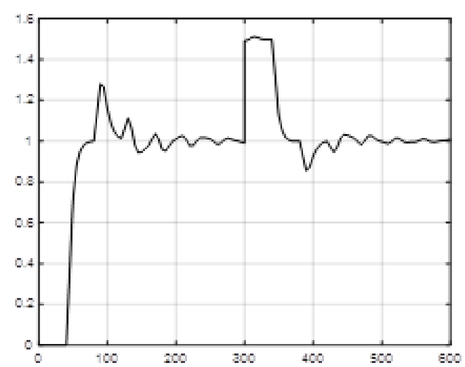

FIGURE 35. YOULA control with filters with mismatch

9. CONCLUSIONS

System outputs have to be controlled, to ensure the required properties for their steady state and transient behaviour considering also the constraints. Control systems are based on negative feedback. Parameters of the controller have to be tuned appropriately to ensure the requested control performance. The paper gives an overview of control structures and control algorithms.

The control algorithms modify the poles of the closed-loop control system to ensure the prescribed dynamics. The control can be realized in continuous or in discrete environment. It is shown that in series control structure with pole cancellation technique only stable processes can be controlled. With state feedback unstable plants can also be controlled and stabilized. In the industry the most widely used algorithms are the *PID* control algorithms. It is shown that YOULA parameterization provides an analytical optimal controller design method ensuring better control performance than *PID* control especially in case of big dead time. The design procedure is very simple requiring only polynomial operations. It can be applied for minimum and non-minimum phase continuous time or discrete processes. This regulator ensures the theoretical best reachable closed-loop property of the control system. It can be shown that the other controller design methods can be considered as special cases of YOULA parameterization.

YOULA controller design is superior to the other controller design methods, as the equations giving the relationships between the input and the output signals are linear in the YOULA parameter Q. For controlling systems with dead time this method gives straightforward solution for controller design. Some important aspects related to improvement of disturbance rejection are also discussed.

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