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EXISTENCE AND ANALYTICITY OF GLOBAL MILD SOLUTIONS TO GMHD EQUATIONS WITH THE CORIOLIS FORCE NEAR AN EQUILIBRIUM

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Abstract. In this paper, we are concerned on the existence and analyticity of global mild solutions to the three-dimensional generalized MHD equations with the Coriolis force in Lei-Lin type space. To be exact, we use the energy method and continuous argument to prove that there exists a global solution near an equilibrium with the initial value $(u_0, b_0) \in \chi^{1-2\alpha}(\mathbb{R}^3) \cap \chi^{1-2\beta}(\mathbb{R}^3)$ for $\frac{1}{2} \leq \alpha, \beta \leq 1$. Moreover, the global solution is analytic for $\frac{1}{2} \leq \alpha = \beta \leq 1$.
Keywords. Analyticity; Generalized MHD equations with the Coriolis force, Global existence of solutions.

1. INTRODUCTION

The three-dimensional generalized MHD equations with the Coriolis force of form is as follows:

$$\begin{cases} \partial_t u + \mu(-\Delta)^{\alpha} u + (u \cdot \nabla) u + \Omega e_3 \times u + \nabla p - (B \cdot \nabla) B = 0, \\ \partial_t B + \nu(-\Delta)^{\beta} B + (u \cdot \nabla) B - (B \cdot \nabla) u = 0, \\ \nabla \cdot u = 0, \nabla \cdot B = 0, \end{cases} \tag{1.1}$$

with the initial condition

$$t = 0 : u = u_0(x),\ B = B_0(x),\ x \in \mathbb{R}^3, \tag{1.2}$$

where $u = u(x,t) \in \mathbb{R}^3$ is the velocity field, $B = B(x,t) \in \mathbb{R}^3$ is magnetic field and $p = p(x,t) \in \mathbb{R}$ is pressure, $\mu > 0$ and $\nu > 0$ are the viscosity coefficients of the flow, $\Omega \in \mathbb{R}$ called the Coriolis parameter represents the speed of rotation around the vertical unit vector $e_3 = (0,0,1)$, $(-\Delta)^{\kappa}$ is defined by $\widehat{(-\Delta)^{\kappa} f} = |\xi|^{2\kappa} \hat{f}$, where $\hat{f}$ is the Fourier transform of f. Coriolis forces describe the deviation of a particle moving in a straight line from a rotating system due to inertia. And the MHD equations with Coriolis force play an non-negligible role in magnetohydrodynamics. In

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recent years, due to its importance, many scholars have studied it and obtained many excellent results.

When $\frac{1}{2} \leq \alpha = \beta \leq 1$, Wang and Wu [17] established the global well-posedness and Gevrey class regularity for the generalized rotating MHD system with the initial data in $\chi^{1-2\alpha}$. And El Baraka and Toumlilin [2] obtained the uniform global well-posedness of the Cauchy problem and discussed the stability of global solutions for the 3D generalized magnetohydrodynamic equations with the Coriolis force, where the initial value was required to be sufficiently small in the critical Fourier-Besov-Morrey spaces. When $\alpha = \beta = 1$, (1.1) is the MHD system with the Coriolis force, which has numerous applications in mathematical geophysics. For details, we refer to [3] and the references therein. Moreover, the MHD system with the Coriolis force was also used in hydromagnetics and [4] magnetohydrodynamic waves in the sun [9]. When $\mu = \nu = 1$, the Cauchy problem of incompressible rotational MHD system was shown by Ahn, Kim, and Lee [1] to be globally well-posed in $H^s(\mathbb{R}^3) \times (L^2 \cap L^q)(\mathbb{R}^3)$ for $\frac{1}{2} < s < \frac{3}{4}$ and $3 < q < \min\{\frac{6}{3-2s}, \frac{27}{6+4s}\}$, which provided that the rotation speed is sufficiently large.

When $\Omega = 0$, the Coriolis force disappears and (1.1) is reduced to the generalized MHD equations. For our purpose, we only recall on some related results in Lei-lin type space. For $\frac{1}{2} \leq \alpha = \beta \leq 1$, Ye [22] and Wang et al [15] obtained the global well-posedness and decay results of the solutions and mild solutions with small initial value, respectively. Liu et al. [12] proved the existence of global solutions to three-dimensional generalized MHD equations with large initial data. In addition to those results mentioned above, when $\frac{1}{2} \leq \alpha \leq \beta \leq 1$, Ye and Zhao [23] obtained global stability and asymptotic decay of zero solutions. Lu, Li and Wang [13] proved analyticity and time-decay rate of global mild solutions near an equilibrium in the critical space $\chi^{1-2\alpha} \cap \chi^{1-2\beta}$. When $\Omega = 0$ and $\alpha = \beta = 1$, (1.1) becomes the classical MHD equations, which has a lot of achievements. For instance, Wang and Wang [19] received global mild solutions under the assumption that the norms of the initial value is precisely bounded by the minimum of the viscosity coefficients. On the basis of [19], Wang [18] obtained asymptotic decay of solutions. Lin et al. [11] proved global smooth solutions for a class of large initial data. More interesting related results can be found in [14] and [20].

When $B = 0$ and $\alpha = 1$, Zhao and Wang [24] concluded that the equations exist uniform global large solutions for a class of special initial value. And in function spaces of Besov type, global well-posedness and ill-posedness of the Navier-Stokes (N-S) equations with the Coriolis force also have been obtained (see [5], [6], [7] and [8]). Besides, based on the Littlewood-Paley theory and Ito integral, Wang and Wu [16] obtained the global existence of stochastic N-S equations with Coriolis force in Fourier-Besov spaces.

Inspired by [10] and [17], the aim of this paper is to study the existence and analyticity of global mild solutions near an equilibrium for the 3D generalized MHD equations with the Coriolis force if $\frac{1}{2} \leq \alpha, \beta \leq 1$. For this purpose, let $b = B - e_l (l = 1,2,3)$. Then the perturbation (u,b) around the equilibrium obeys

$$\begin{cases} \partial_t u + \mu(-\Delta)^{\alpha} u + (u \cdot \nabla)u + \Omega e_3 \times u + \nabla p - (b \cdot \nabla)b - \partial_l b = 0, \\ \partial_t b + \nu(-\Delta)^{\beta} b + (u \cdot \nabla)b - (b \cdot \nabla)u - \partial_l u = 0, \\ \nabla \cdot u = 0, \nabla \cdot b = 0 \end{cases} \tag{1.3}$$

with the initial condition

$$t=0: \quad u=u_0(x), \quad b=b_0(x), \quad x\in\mathbb{R}^3. \tag{1.4}$$

In what follows, we state our main result as following.

Theorem 1.1. *Let $\frac{1}{2}\le\alpha\le\beta\le 1$. Assume that $u_0,b_0\in\chi^{1-2\alpha}(\mathbb{R}^3)\cap\chi^{1-2\beta}(\mathbb{R}^3)$ and put*

$$\delta=\|(u_0,b_0)\|_{\chi^{1-2\alpha}}+\|(u_0,b_0)\|_{\chi^{1-2\beta}}.$$

Then there exists $\delta_0>0$ such that if $\delta<\delta_0$, then problem (1.3)-(1.4) has a global solution (u,b). Furthermore, for any $t>0$, it holds that

$$\|(u,b)(t)\|_{\chi^{1-2\alpha}}+\|(u,b)(t)\|_{\chi^{1-2\beta}}+\omega\int_0^t\|(u,b)(\tau)\|_{\chi^1}d\tau\le C\delta, \tag{1.5}$$

where $\omega=\min\{\mu,\nu\}$.

Theorem 1.2. *Assume that $\alpha=\beta$ and the conditions of Theorem 1.1 hold. Then the solution is analytic in the sense that*

$$\|e^{\omega|D|^\alpha\sqrt{t}}(u,b)(t)\|_{\chi^{1-2\alpha}}+\int_0^t\|e^{\omega|D|^\alpha\sqrt{t}}(u,b)(\tau)\|_{\chi^1}d\tau\le C\|(u_0,b_0)\|_{\chi^{1-2\alpha}}, \tag{1.6}$$

where $e^{\omega|D|^\alpha\sqrt{t}}$ is a Fourier multiplier whose symbol is given by $e^{\omega|\xi|^\alpha\sqrt{t}}$ and $\omega=\min\{\mu,\nu\}$.

Remark 1.3. In [17], Wang and Wu obtained the global stability of the zero solution when $\frac{1}{2}\le\alpha=\beta\le 1$. The biggest difference of Theorem 1.1 is that the terms $\partial_l u$ and $\partial_l b$ appear in this paper due to the perturbations near the steady solution $(0,e_l)$ and $\frac{1}{2}\le\alpha,\beta\le 1$.

Remark 1.4. On the one hand, owing to the existence of linear terms $e_3\times u$, $\partial_l u$, and $\partial_l b$, it is very difficult to deduce the solutions operator and decay property of the solutions operator to the problem (1.3)-(1.4). Based on the solutions operator and decay property of the solutions operator, the existence and analyticity of global solutions to three-dimensional GMHD equations have been proved in [13] and [21]. Fortunately, this difficulty can be overcome by the energy methods and frequency division technology in the Fourier space.

Notations

Finally, we introduce some notations used in this paper. The Fourier transform f is defined by

$$\widehat{f}(\xi)=\int_{\mathbb{R}^3}f(x)e^{-ix\cdot\xi}dx.$$

The Lei-Lin type space χ^s is defined by

$$\chi^s:=\left\{f\in\mathscr{D}'(\mathbb{R}^3)\middle|\int_{\mathbb{R}^3}|\xi|^s|\widehat{f}(\xi)|d\xi<\infty, s\in\mathbb{R}\right\}$$

and the association norm is given by

$$\|f\|_{\chi^s}=\int_{\mathbb{R}^3}|\xi|^s|\widehat{f}(\xi)|d\xi<\infty.$$

2. Global Mild Solutions

In this section, we are ready to prove Theorem 1.1, for the global existence of mild solutions to the 3D incompressible generalized MHD equations with Coriolis force in Lei-Lin type space. To prove that there exists global mild solutions to the problem (1.3)-(1.4), we introduce the following Lemma.

Lemma 2.1. *(see [22]) Assume that* $\frac{1}{2}\le\gamma\le 1$. *Then the following inequality*

$$|\xi|^{2(1-\gamma)}\le 2^{1-2\gamma}(|\eta||\xi-\eta|^{1-2\gamma}+|\xi-\eta||\eta|^{1-2\gamma})$$

holds for any $\xi,\eta\in\mathbb{R}^n$.

Next, we give the proof of Theorem 1.1.

Proof. To prove the theorem 1.1, we first define the following two quantities

$$X(t)=\sup_{0\le\tau\le t}(\|(u,b)(\tau)\|_{\chi^{1-2\alpha}}+\|(u,b)(\tau)\|_{\chi^{1-2\beta}}),\ \ Y(t)=\omega\int_0^t\|(u,b)(\tau)\|_{\chi^1}d\tau. \tag{2.1}$$

We apply Fourier transform to (1.3), (1.4) and obtain

$$\begin{cases}\partial_t\widehat{u}+\mu|\xi|^{2\alpha}\widehat{u}+\widehat{\Omega e_3\times u}+i\xi\widehat{p}-i\xi_3\widehat{b}=-\nabla\cdot(\widehat{u\otimes u-b\otimes b}),\\ \partial_t\widehat{b}+\nu|\xi|^{2\beta}\widehat{b}-i\xi_3\widehat{u}=-\nabla\cdot(\widehat{u\otimes b-b\otimes u}),\\ \xi\cdot\widehat{u}=0,\ \xi\cdot\widehat{b}=0,\\ t=0:\widehat{u}=\widehat{u}_0(\xi),\ \widehat{b}=\widehat{b}_0(\xi).\end{cases} \tag{2.2}$$

From the first, second and third equations in (2.2), we obtain

$$\begin{cases}\partial_t|\widehat{u}|^2+2\mu|\xi|^{2\alpha}|\widehat{u}|^2+2Re\left\{[(\widehat{\Omega e_3\times u})-i\xi_3\widehat{b}]\cdot\bar{\widehat{u}}\right\}=-2Re\left\{\nabla\cdot(\widehat{u\otimes u-b\otimes b})\cdot\bar{\widehat{u}}\right\},\\ \partial_t|\widehat{b}|^2+2\nu|\xi|^{2\beta}|\widehat{b}|^2-2Re\left\{i\xi_3\widehat{u}\cdot\bar{\widehat{b}}\right\}=-2Re\left\{\nabla\cdot(\widehat{u\otimes b-b\otimes u})\cdot\bar{\widehat{b}}\right\}.\end{cases} \tag{2.3}$$

By direct computation, we arrive at

$$2Re\left\{(\widehat{\Omega e_3\times u})\cdot\bar{\widehat{u}}\right\}=2Re\left\{\Omega\left(-\widehat{u}_2\bar{\widehat{u}}_1+\widehat{u}_1\bar{\widehat{u}}_2\right)\right\}=\Omega\left(-\widehat{u}_2\bar{\widehat{u}}_1+\widehat{u}_1\bar{\widehat{u}}_2-\bar{\widehat{u}}_2\widehat{u}_1+\bar{\widehat{u}}_1\widehat{u}_2\right)=0 \tag{2.4}$$

and

$$2Re\left\{i\xi_3\widehat{b}\cdot\bar{\widehat{u}}\right\}+2Re\left\{i\xi_3\widehat{u}\cdot\bar{\widehat{b}}\right\}=2Re\left\{i\xi_3\left(\widehat{b}\cdot\bar{\widehat{u}}+\widehat{u}\cdot\bar{\widehat{b}}\right)\right\}=4Re\left\{i\xi_3Re\left\{\widehat{b}\cdot\bar{\widehat{u}}\right\}\right\}=0. \tag{2.5}$$

It follows from (2.3)-(2.5) that

$$\begin{aligned}\partial_t(|\widehat{u}|^2+|\widehat{b}|^2)+2(\mu|\xi|^{2\alpha}|\widehat{u}|^2+\nu|\xi|^{2\beta}|\widehat{b}|^2)\ &=\ -2Re\left\{\nabla\cdot(\widehat{u\otimes u-b\otimes b})\cdot\bar{\widehat{u}}\right\}\\ &\quad -2Re\left\{\nabla\cdot(\widehat{u\otimes b-b\otimes u})\cdot\bar{\widehat{b}}\right\}.\end{aligned} \tag{2.6}$$

If $\varepsilon>0$, then

$$\partial_t(|\widehat{u}|^2+|\widehat{b}|^2)=\partial_t\left(|\widehat{u}|^2+|\widehat{b}|^2+\varepsilon^2\right)=2\sqrt{|\widehat{u}|^2+|\widehat{b}|^2+\varepsilon^2}\,\partial_t\sqrt{|\widehat{u}|^2+|\widehat{b}|^2+\varepsilon^2}.$$

Due to (2.6) and the above equality, we can obtain

$$\begin{aligned}
&\partial_t\sqrt{|\widehat{u}|^2+|\widehat{b}|^2+\varepsilon^2}+\frac{\mu|\xi|^{2\alpha}|\widehat{u}|^2+\nu|\xi|^{2\beta}|\widehat{b}|^2}{\sqrt{|\widehat{u}|^2+|\widehat{b}|^2+\varepsilon^2}}\\
\leq\ & \left(\left|\nabla\cdot(\widehat{u\otimes u-b\otimes b})\right|\right)\frac{|\widehat{u}|}{\sqrt{|\widehat{u}|^2+|\widehat{b}|^2+\varepsilon^2}}+\left(\left|\nabla\cdot(\widehat{u\otimes b-b\otimes u})\right|\right)\frac{|\widehat{b}|}{\sqrt{|\widehat{u}|^2+|\widehat{b}|^2+\varepsilon^2}}\\
\leq\ & \left|\nabla\cdot(\widehat{u\otimes u-b\otimes b})\right|+\left|\nabla\cdot(\widehat{u\otimes b-b\otimes u})\right|.
\end{aligned}$$

Note that

$$\omega\min\left\{|\xi|^{2\alpha},|\xi|^{2\beta}\right\}\frac{|\hat{u}|^2+|\hat{b}|^2}{\sqrt{|\hat{u}|^2+|\hat{b}|^2+\varepsilon^2}}\leq\frac{\mu|\xi|^{2\alpha}|\widehat{u}|^2+\nu|\xi|^{2\beta}|\widehat{b}|^2}{\sqrt{|\widehat{u}|^2+|\widehat{b}|^2+\varepsilon^2}}.$$

Combining (2.7) with the above inequality and letting $\varepsilon\to 0$ in resulting inequality, we have

$$\begin{aligned}
&\partial_t\sqrt{|\widehat{u}|^2+|\widehat{b}|^2}+\omega\min\left\{|\xi|^{2\alpha},|\xi|^{2\beta}\right\}\sqrt{|\widehat{u}|^2+|\widehat{b}|^2}\\
\leq\ & \left|\nabla\cdot(\widehat{u\otimes u-b\otimes b})\right|+\left|\nabla\cdot(\widehat{u\otimes b-b\otimes u})\right|.
\end{aligned}\tag{2.7}$$

Noting that $\sqrt{|\widehat{u}|^2+|\widehat{b}|^2}\geq\frac{\sqrt{2}}{2}(|\widehat{u}|+|\widehat{b}|)$, we obtain from (2.7) that

$$\begin{aligned}
&\partial_t(|\widehat{u}|+|\widehat{b}|)+\omega\min\left\{|\xi|^{2\alpha},|\xi|^{2\beta}\right\}(|\widehat{u}|+|\widehat{b}|)\\
\leq\ & \sqrt{2}\Big(\left|\nabla\cdot(\widehat{u\otimes u-b\otimes b})\right|+\left|\nabla\cdot(\widehat{u\otimes b-b\otimes u})\right|\Big).
\end{aligned}\tag{2.8}$$

Integrating (2.8) with respect to t, we obtain

$$\begin{aligned}
&|\widehat{u}|+|\widehat{b}|+\omega\int_0^t\min\left\{|\xi|^{2\alpha},|\xi|^{2\beta}\right\}(|\widehat{u}|+|\widehat{b}|)(\xi,\tau)d\tau\\
\leq & C(|\widehat{u}_0|+|\widehat{b}_0|)+C\int_0^t\Big(\left|\nabla\cdot(\widehat{u\otimes u-b\otimes b})\right|+\left|\nabla\cdot(\widehat{u\otimes b-b\otimes u})\right|\Big)(\xi,\tau)d\tau.
\end{aligned}\tag{2.9}$$

Due to $\frac{1}{2}\leq\alpha\leq\beta\leq 1$, we have

$$\min\left\{|\xi|^{2\alpha},|\xi|^{2\beta}\right\}=\begin{cases}|\xi|^{2\beta}, & |\xi|\leq 1;\\ |\xi|^{2\alpha}, & |\xi|>1.\end{cases}\tag{2.10}$$

Multiplying (2.9) by $|\xi|^{1-2\alpha}$ and integrating the resulting equation, we obtain from (2.10) that

$$
\begin{aligned}
&\|(u,b)(t)\|_{\chi^{1-2\alpha}}+\omega\int_0^t\int_{|\xi|\le 1}|\xi|^{2\beta}|\xi|^{1-2\alpha}(|u(\tau)|+|b(\tau)|)d\xi d\tau\\
&+\omega\int_0^t\int_{|\xi|>1}|\xi|^{2\alpha}|\xi|^{1-2\alpha}(|u(\tau)|+|b(\tau)|)d\xi d\tau\\
\le\;& C\|(u_0,b_0)\|_{\chi^{1-2\alpha}}+C\int_0^t\int_{\mathbb{R}^3}|\xi|^{2-2\alpha}\left|(u\otimes\widehat{u-b}\otimes b)\right|d\xi d\tau\\
&+C\int_0^t\int_{\mathbb{R}^3}|\xi|^{2-2\alpha}\left|(u\otimes\widehat{b-b}\otimes u)\right|d\xi d\tau \qquad (2.11)\\
\le\;& C\|(u_0,b_0)\|_{\chi^{1-2\alpha}}+C\int_0^t\int_{\mathbb{R}^3}|\xi|^{2-2\alpha}(\left|\widehat{u\otimes u}\right|+\left|\widehat{b\otimes b}\right|+\left|\widehat{u\otimes b}\right|+\left|\widehat{b\otimes u}\right|)d\xi d\tau.
\end{aligned}
$$

By multiplying (2.9) by $|\xi|^{1-2\beta}$ in the same way, we arrive at

$$
\begin{aligned}
&\|(u,b)(t)\|_{\chi^{1-2\beta}}+\omega\int_0^t\int_{|\xi|\le 1}|\xi|^{2\beta}|\xi|^{1-2\beta}(|u(\tau)|+|b(\tau)|)d\xi d\tau\\
&+\omega\int_0^t\int_{|\xi|>1}|\xi|^{2\alpha}|\xi|^{1-2\beta}(|u(\tau)|+|b(\tau)|)d\xi d\tau\\
\le\;& C\|(u_0,b_0)\|_{\chi^{1-2\beta}}+C\int_0^t\int_{\mathbb{R}^3}|\xi|^{2-2\beta}\left|(u\otimes\widehat{u-b}\otimes b)\right|d\xi d\tau\\
&+C\int_0^t\int_{\mathbb{R}^3}|\xi|^{2-2\beta}\left|(u\otimes\widehat{b-b}\otimes u)\right|d\xi d\tau \qquad (2.12)\\
\le\;& C\|(u_0,b_0)\|_{\chi^{1-2\beta}}+C\int_0^t\int_{\mathbb{R}^3}|\xi|^{2-2\beta}(\left|\widehat{u\otimes u}\right|+\left|\widehat{b\otimes b}\right|+\left|\widehat{u\otimes b}\right|+\left|\widehat{b\otimes u}\right|)d\xi d\tau.
\end{aligned}
$$

Adding (2.11) and (2.12), we have

$$
\begin{aligned}
&\|(u,b)(t)\|_{\chi^{1-2\alpha}}+\|(u,b)(t)\|_{\chi^{1-2\beta}}+\omega\int_0^t\|(u,b)(\tau)\|_{\chi^1}d\tau\\
\le\;& C(\|(u_0,b_0)\|_{\chi^{1-2\alpha}}+\|(u_0,b_0)\|_{\chi^{1-2\beta}}) \qquad (2.13)\\
&+C\int_0^t\int_{\mathbb{R}^3}(|\xi|^{2-2\alpha}+|\xi|^{2-2\beta})(\left|\widehat{u\otimes u}\right|+\left|\widehat{b\otimes b}\right|+\left|\widehat{u\otimes b}\right|+\left|\widehat{b\otimes u}\right|)d\xi d\tau.
\end{aligned}
$$

It follows from Lemma 2.1 and Young inequality that

$$
\begin{aligned}
&\int_0^t \int_{\mathbb{R}^3} (|\xi|^{2-2\alpha}+|\xi|^{2-2\beta}) \left|\widehat{u\otimes b}\right| (\xi,\tau) d\xi d\tau \\
\leq\ & C\int_0^t \int_{\mathbb{R}^3\times\mathbb{R}^3} \left[(|\eta|^{1-2\alpha}|\xi-\eta|+|\eta||\xi-\eta|^{1-2\alpha})|\hat{u}(\eta)\otimes\hat{b}(\xi-\eta)|d\eta\right] d\xi d\tau \\
&+C\int_0^t \int_{\mathbb{R}^3\times\mathbb{R}^3} \left[|\eta|^{1-2\beta}|\xi-\eta|+|\eta||\xi-\eta|^{1-2\beta})|\hat{u}(\eta)\otimes\hat{b}(\xi-\eta)|d\eta\right] d\xi d\tau \\
\leq\ & C\int_0^t \left[\int_{\mathbb{R}^3} |\eta|^{1-2\alpha}|\hat{u}(\eta)|d\eta \int_{\mathbb{R}^3} |\xi-\eta||\hat{b}(\xi-\eta)|d\xi\right] d\tau \qquad (2.14)\\
&+C\int_0^t \left[\int_{\mathbb{R}^3} |\xi-\eta|^{1-2\alpha}|\hat{b}(\xi-\eta)|d\xi \int_{\mathbb{R}^3} |\eta||\hat{u}(\eta)|d\eta\right] d\tau \\
&+C\int_0^t \left[\int_{\mathbb{R}^3} |\xi-\eta|^{1-2\beta}|\hat{b}(\xi-\eta)|d\xi \int_{\mathbb{R}^3} |\eta||\hat{u}(\eta)|d\eta\right] d\tau \\
&+C\int_0^t \left[\int_{\mathbb{R}^3} |\eta|^{1-2\beta}|\hat{u}(\eta)|d\eta \int_{\mathbb{R}^3} |\xi-\eta||\hat{b}(\xi-\eta)|d\xi\right] d\tau \\
\leq\ & C\int_0^t \left(\|u(\tau)\|_{\chi^{1-2\alpha}}\|b(\tau)\|_{\chi^1}+\|b(\tau)\|_{\chi^{1-2\alpha}}\|u(\tau)\|_{\chi^1}\right) d\tau \\
&+C\int_0^t \left(\|u(\tau)\|_{\chi^{1-2\beta}}\|b(\tau)\|_{\chi^1}+\|b(\tau)\|_{\chi^{1-2\beta}}\|u(\tau)\|_{\chi^1}\right) d\tau \\
\leq\ & CX(t)Y(t).
\end{aligned}
$$

The same procedure entails that

$$
\begin{aligned}
&\int_0^t \int_{\mathbb{R}^3} (|\xi|^{2-2\alpha}+|\xi|^{2-2\beta}) (\left|\widehat{u\otimes u}\right|+\left|\widehat{b\otimes b}\right|+\left|\widehat{b\otimes u}\right|)(\xi,\tau) d\xi d\tau \\
\leq\ & C\int_0^t \Big(\|u(\tau)\|_{\chi^{1-2\alpha}\cap\chi^{1-2\beta}}\|u(\tau)\|_{\chi^1}+\|b(\tau)\|_{\chi^{1-2\alpha}\cap\chi^{1-2\beta}}\|b(\tau)\|_{\chi^1} \qquad (2.15)\\
&+\|u(\tau)\|_{\chi^{1-2\alpha}\cap\chi^{1-2\beta}}\|b(\tau)\|_{\chi^1}+\|b(\tau)\|_{\chi^{1-2\alpha}\cap\chi^{1-2\beta}}\|u(\tau)\|_{\chi^1}\Big) d\tau \\
\leq\ & CX(t)Y(t).
\end{aligned}
$$

Inserting (2.14) and (2.15) into (2.13) yields

$$
\begin{aligned}
&\|(u,b)(t)\|_{\chi^{1-2\alpha}}+\|(u,b)(t)\|_{\chi^{1-2\beta}}+\omega\int_0^t (\|u(\tau)\|_{\chi^1}+\|b(\tau)\|_{\chi^1})d\tau \\
\leq\ & C(\|(u_0,b_0)\|_{\chi^{1-2\alpha}}+\|(u_0,b_0)\|_{\chi^{1-2\beta}})+CX(t)Y(t),
\end{aligned}
$$

which implies that

$$
X(t)+Y(t)\leq C_0(\|(u_0,b_0)\|_{\chi^{1-2\alpha}}+\|(u_0,b_0)\|_{\chi^{1-2\beta}})+\frac{1}{2}C_1(X(t)+Y(t))^2. \qquad (2.16)
$$

Next, we complete the final proof by using continuous argument. To this end, we suppose that

$$
X(t)+Y(t)\leq 4C_0\delta. \qquad (2.17)
$$

If we take δ sufficiently small such that $\delta\leq\frac{1}{8C_0C_1}$, then we may derive from (2.15) and (2.17)

$$
X(t)+Y(t)\leq 2C_0\delta. \qquad (2.18)
$$

By the continuous argument, (1.3), (1.4) admits a global solution $(u,b)\in C([0,\infty);\chi^{1-2\alpha}\cap\chi^{1-2\beta}(\mathbb{R}^3))\bigcap L^1([0,\infty);\chi^1(\mathbb{R}^3))$. We complete the proof of Theorem 1.1. $\square$

3. The Proof of Theorem 1.2

In this section, based on the existence of the global solution, we prove the analyticity of global solutions for problem (1.3)-(1.4). we give the proof of Theorem 1.2 as follows.

Proof. Multiplying (2.8) by $e^{\omega|\xi|^{2\alpha}t}$, we obtain

$$\begin{aligned}&\frac{d}{dt}\left[e^{\omega|\xi|^{2\alpha}t}(|\hat{u}|+|\hat{b}|)\right]\\ \leq&\sqrt{2}e^{\omega|\xi|^{2\alpha}t}\left(\left|\nabla\cdot(u\widehat{\otimes u-b}\otimes b)\right|+\left|\nabla\cdot(u\widehat{\otimes b-b}\otimes u)\right|\right).\end{aligned}\tag{3.1}$$

Integrating (3.1) with respect to t, we have

$$\begin{aligned}e^{\omega|\xi|^{2\alpha}t}(|\hat{u}|+|\hat{b}|)\leq&|\hat{u}_0|+|\hat{b}_0|+\sqrt{2}\int_0^t e^{\omega|\xi|^{2\alpha}\tau}\left|\nabla\cdot(u\widehat{\otimes u-b}\otimes b)\right|d\tau\\&+\sqrt{2}\int_0^t e^{\omega|\xi|^{2\alpha}\tau}\left|\nabla\cdot(u\widehat{\otimes b-b}\otimes u)\right|d\tau,\end{aligned}\tag{3.2}$$

which implies that

$$\begin{aligned}|\hat{u}|+|\hat{b}|\leq&e^{-\omega|\xi|^{2\alpha}t}(|\hat{u}_0|+|\hat{b}_0|)+\sqrt{2}\int_0^t e^{-\omega|\xi|^{2\alpha}(t-\tau)}\left|\nabla\cdot(u\widehat{\otimes u-b}\otimes b)\right|d\tau\\&+\sqrt{2}\int_0^t e^{-\omega|\xi|^{2\alpha}(t-\tau)}\left|\nabla\cdot(u\widehat{\otimes b-b}\otimes u)\right|d\tau.\end{aligned}\tag{3.3}$$

Note that $|\xi|^\alpha\leq|\xi-\eta|^\alpha+|\eta|^\alpha$. Multiplying (3.3) by $e^{\omega|\xi|^\alpha\sqrt{t}}$ and letting $\hat{\mathscr{U}}=e^{\omega|\xi|^\alpha\sqrt{t}}\hat{u}$ and $\hat{\mathscr{B}}=e^{\omega|\xi|^\alpha\sqrt{t}}\hat{b}$, we have

$$\begin{aligned}|\hat{\mathscr{U}}|+|\hat{\mathscr{B}}|\leq&e^{-\omega|\xi|^{2\alpha}t}e^{\omega|\xi|^\alpha\sqrt{t}}(|\hat{u}_0|+|\hat{b}_0|)+\sqrt{2}\int_0^t e^{-\omega|\xi|^{2\alpha}(t-\tau)}|\xi|\left|\widehat{\mathscr{U}\otimes\mathscr{U}}\right|d\tau\\&+\sqrt{2}\int_0^t e^{-\omega|\xi|^{2\alpha}(t-\tau)}|\xi|\left(\left|\widehat{\mathscr{B}\otimes\mathscr{B}}\right|+\left|\widehat{\mathscr{U}\otimes\mathscr{B}}\right|+\left|\widehat{\mathscr{B}\otimes\mathscr{U}}\right|\right)d\tau.\end{aligned}\tag{3.4}$$

Noting that $e^{-\omega|\xi|^{2\alpha}t+\omega|\xi|^\alpha\sqrt{t}}$ is uniformly bounded in time. Multiplying (3.4) by $|\xi|^{1-2\alpha}$, integrating with respect to ξ and then using the Lemma 2.1, we obtain

$$\begin{aligned}&\|(\mathscr{U},\mathscr{B})\|_{\chi^{1-2\alpha}}\leq C\|(u_0,b_0)\|_{\chi^{1-2\alpha}}+C\int_0^t\big(\|\mathscr{U}\|_{\chi^{1-2\alpha}}\|\mathscr{U}\|_{\chi^1}+\|\mathscr{B}\|_{\chi^{1-2\alpha}}\|\mathscr{B}\|_{\chi^1}+\\&\|\mathscr{U}\|_{\chi^{1-2\alpha}}\|\mathscr{B}\|_{\chi^1}+\|\mathscr{B}\|_{\chi^{1-2\alpha}}\|\mathscr{U}\|_{\chi^1}\big)d\tau.\end{aligned}\tag{3.5}$$

Next, letting (3.4) multiply by $|\xi|$ and integrating the results with respect to ξ and t, respectively, we see that

$$\begin{aligned}
&\int_0^t \|(\mathscr{U},\mathscr{B})\|_{\chi^1} d\tau \le \int_0^t \int_{\mathbb{R}^3} e^{-\omega|\xi|^{2\alpha}\tau}|\xi|(|\hat{u}_0|+|\hat{b}_0|)d\xi d\tau \\
&+C\int_s^t\int_0^s\int_{\mathbb{R}^3} e^{-\omega|\xi|^{2\alpha}(s-\tau)}|\xi|^2\left(\left|\widehat{\mathscr{U}\otimes\mathscr{U}}\right|+\left|\widehat{\mathscr{B}\otimes\mathscr{B}}\right|+\left|\widehat{\mathscr{U}\otimes\mathscr{B}}\right|+\left|\widehat{\mathscr{B}\otimes\mathscr{U}}\right|\right)d\xi d\tau ds \\
\le& \int_0^t e^{-\omega|\xi|^{2\alpha}\tau}|\xi|^{2\beta}d\tau\int_{\mathbb{R}^3}|\xi|^{1-2\alpha}(|\hat{u}_0|+|\hat{b}_0|)d\xi \\
&+C\int_s^t e^{-\omega|\xi|^{2\alpha}(s-\tau)}|\xi|^{2\alpha}ds\int_0^t\int_{\mathbb{R}^3}|\xi|^{2-2\alpha}\Big(\left|\widehat{\mathscr{U}\otimes\mathscr{B}}\right|+\left|\widehat{\mathscr{B}\otimes\mathscr{U}}\right| \\
&\quad+\left|\widehat{\mathscr{U}\otimes\mathscr{U}}\right|+\left|\widehat{\mathscr{B}\otimes\mathscr{B}}\right|\Big)d\xi d\tau \\
&\le C\|(u_0,b_0)\|_{\chi^{1-2\alpha}}+C\int_0^t \|(\mathscr{U},\mathscr{B})\|_{\chi^{1-2\alpha}}\|(\mathscr{U},\mathscr{B})\|_{\chi^1}d\tau.
\end{aligned} \tag{3.6}$$

Summing (3.5) and (3.6), we arrive at

$$\begin{aligned}
&\|(\mathscr{U},\mathscr{B})\|_{\chi^{1-2\alpha}}+\int_0^t \|(\mathscr{U},\mathscr{B})\|_{\chi^1}d\tau \\
\le& C\|(u_0,b_0)\|_{\chi^{1-2\alpha}}+C\int_0^t \|(\mathscr{U},\mathscr{B})\|_{\chi^{1-2\alpha}}\|(\mathscr{U},\mathscr{B})\|_{\chi^1}d\tau.
\end{aligned}$$

By the continuous argument, (1.6) is established. We complete the proof of Theorem 1.2. □

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