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ON THE SYNTHESIS OF OPTIMAL DYNAMIC SYSTEMS UNDER CONDITION OF UNCERTAINTY

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Dedicated to the memory of Professor Rafail Gabasov

Abstract. The problem of synthesis of linear optimal dynamical systems (classical feedbacks, open loop and closed loop) is considered. The linear terminal problem of controlling a dynamic system under the conditions of constantly acting disturbances is examined in the class of discrete control actions. The method of constructing the current values of the guaranteed open and closed loops in real time, based on the dual method of linear programming is described.

Keywords. Guaranteed controls; Optimal control; Open and closed loops; Real-time control; Uncertainty.

1. Introduction

The development of digital computing technology has led to the discovery of a new control principle — real-time control. This principle does not require the preliminary construction of optimal feedback in an analytical or tabulated form. Its use sharply reduces the requirements for computer memory. The current control values required for the control are calculated in real time during each particular control process. At the same time, ultimate success largely depends on the number and speed of digital computing devices, as well as on the effectiveness of the optimization methods used.

The principle of real-time control, according to which no preliminary construction of feedbacks is required, and their desired values are calculated in real time during the control processes. The success of computer technology and the constructive theory of extreme problems contributed to the implementation of the new control principle. The introduction of the modern control principle was hindered by the lack of fast algorithms for calculating optimal controls.

The methods of real-time optimal control has been developed since early 90-s of the past century in Minsk. Some of the results of this work were described in [1, 2, 6, 7].

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2. Classical Optimal State Feedbacks

Let $T=[t_*,t^*]$ be a time interval, $-\infty<t_*<t^*<\infty$; $h=(t^*-t_*)/N$ — quantification period of time; $N>1$ — natural number; $T_h=\{t_*,t_*+h,\ldots,t^*-h\}$; $T(\tau)=[\tau,t^*]$, $t_*\leq\tau<t^*$; $A(t)\in\mathbb{R}^{n\times n}$, $b(t)\in\mathbb{R}^n$, $m(t)\in\mathbb{R}^n$, $t\in T$ — piecewise continuous functions; $c\in\mathbb{R}^n$;

$$X^*=\{x\in\mathbb{R}^n: g_*\leq Hx\leq g^*\},\ g^*,\ g_*\in\mathbb{R}^m;\ x_0\in\mathbb{R}^n;$$

$h_i\in\mathbb{R}^n$ — i-row of matrix $H\in\mathbb{R}^{m\times n}$, $i\in I=\{1,2,\ldots,m\}$; $x(\cdot)=\big(x(t),t\in T\big)$;

$$U=\{u\in\mathbb{R}:|u|\leq L_u\},\ 0<L_u<\infty;\quad W=\{w\in\mathbb{R}:|w|\leq L_w\},\ 0<L_w<\infty.$$

Definition 2.1. The function $u(\cdot)$ is called discrete (with quantization period h) if

$$u(t)=u(\tau),\ t\in[\tau,\tau+h],\ \tau\in T_h.$$

Let a linear optimal control problem be considered in the class of discrete control actions

$$c'x(t^*)\to\max;\quad \dot{x}(t)=A(t)x(t)+b(t)u(t),\ x(t_*)=x_0,\ x(t^*)\in X^*,\ u(t)\in U,\ t\in T,\quad (2.1)$$

where $x=x(t)\in\mathbb{R}^n$ is the state of the control object at time t, and $u=u(t)\in\mathbb{R}$ is the value of the control at time t.

Definition 2.2. The control $u(\cdot)$ is called available if it is piecewise continuous functions and $u(t)\in U$, $t\in T$.

It is clear that every available control $u(\cdot)$ corresponds to a single trajectory $x(\cdot)$ of dynamic system (2.1).

Definition 2.3. The available control $u(\cdot)$ is a program if $x(t^*)\in X^*$.

Definition 2.4. A program $u^0(t)$, $t\in T$ is called optimal (the program solution of problem (2.1)) if on the corresponding trajectory $x^0(\cdot)$ the quality of the program $J\big(u(\cdot)\big)=c'x(t^*)$ attains its maximum value on the set of all programs

$$J(u^0(\cdot)=\max J\big(u(\cdot)\big),\ u(t)\in U,\ t\in T.$$

Assume that at each current moment $\tau\in T_h$ of the control process the current state $x^*(\tau)$ of the dynamic system (2.1) will be exactly known. Accordingly, we will insert problem (2.1) into the set of problems:

$$c'x(t^*)\to\max;\ \dot{x}(t)=A(t)x(t)+b(t)u(t),\ x(\tau)=z,\ x(t^*)\in X^*,\ u(t)\in U,\ t\in T(\tau),\quad (2.2)$$

depending upon the position.

Suppose $u^0(t|\tau,z)$, $t\in T(\tau)$, is the optimal program for the problem (2.2) for the position (τ,z), and X_τ is the set of states z for which this problem has a solution.

Definition 2.5. The function

$$u^0(\tau,z)=u^0(\tau|\tau,z),\ z\in X_\tau,\ \tau\in T_h,\quad (2.3)$$

is called optimal discrete state feedback or a positional solution of the problem.

The substitution function (2.3) into (2.1) (system closure) leads to an optimal control system:

$$\dot{x}(t)=A(t)x(t)+b(t)u^0(t,x),\ x(t_*)=x_0,\quad (2.4)$$

which is the aim point of optimal synthesis. This aim in the theory of the maximum principle was achieved by constructing switching surfaces [14]. In dynamic programming, feedback (2.3) was obtained from solutions of Bellman equations [5]. In the class of control actions adopted above, equation (2.4) always has a unique solution $x(\cdot)$, which is constructed incrementally. None of the known implementation of optimal feedbacks using the dynamic programming method allows to overcome its main drawback, which was called by Bellman the "curse of dimensionality". The amount of computer memory required grows exponentially relative to the dimension of the states of the control system to the dimension of the control system states. Approximations of the Bellman function by finite-parametric functions, interpolation, and others do not solve this problem either. The fact is that dynamic programming based on the classical closed-loop control principle requires building connections before the start of the control process. The latter principle arose before the modern scientific and technological revolution and could not use the achievements in technology and mathematics of that time. One of the results of the scientific and technological revolution is a real-time control.

Another method of optimal synthesis is described below. The function (2.3) is not constructed in advance, while its necessary values are calculated during the control process. To clarify the essence of the new approach, we assume that function (2.3) is constructed. It is construct on a deterministic model and intended to control a dynamic object though whose behavior $x^*(\cdot)$ generally differs from the behavior of the $x(\cdot)$ model. Let we analyze its use in a concrete process of control action a dynamic object, which is affected by an unknown perturbation $w^*(\cdot)$. This perturbation will generate a transient in the closed loop system (2.4), which is measurable and satisfies the equation:

$$\dot{x}^*(t) = A(t)x^*(t) + b(t)u^0(t, x^*(t)) + w^*(t),\ t \in T. \tag{2.5}$$

It is clear that the optimal feedback is partially used in the control process covered, and it is enough to know only its values along the isolated curve $x^*(\cdot)$.

Let the control process start at the moment t_* with the input of the control object

$$u^*(t) = u^0(t_*, x_0),\ t \geq t_*.$$

At the time $t = t_* + h$, the state of the object $x^*(t_* + h)$ becomes known. Due to this information for the time $s(t_* + h)$, $0 \leq s(t_* + h) < h$ a new value of the control $u^0\big(t_* + h, x^*(t_* + h)\big)$ is found. From the moment $t_* + h + s(t_* + h)$, the input of the object is affected by control

$$u^*(t) = u^0\big(t_* + h, x^*(t_* + h)\big),\ t \geq t_* + h + s(t_* + h),$$

where $s(t_* + h)$ is the search time for the value $u^0\big(t_* + h + s(t_* + h), x^*(t_* + h)\big)$.

To this end, a sequence of control actions is obtained

$$u^*(t) = u^0\big(\tau, x^*(\tau)\big),\ t \in [\tau + s(\tau), \tau + h + s(\tau + h)],\ 0 \leq s(\tau) < h,\ \tau \in T_h,$$

which called the realization of optimal feedback (2.3) in a concrete process of control.

Definition 2.6. Let us call the optimal Regulator a device that for each current position $(\tau, x^*(\tau))$ is capable of calculating the value of $u^*(t)$, $t \in T$ in a time not exceeding h.

According to Definition 2.5, the value of the control action $u^0(t_*, x_0)$ is equal to the initial value of the optimal program for the position (t_*, x_0), which can be found before the start of the control process. The next value of the control action $u^0\big(t_* + h, x^*(t_* + h)\big)$ is obtained from the optimal program $u^0(t_*, x_0)$ by the dual linear programming method by correcting the previous optimal support [2, 8, 9].

An algorithm for the operation of an optimal Regulator was described in [8] and was based on: 1) the discreteness of control actions; 2) the reduction of the problem of constructing a sequence of optimal programs to linear programming problems (LP); 3) the dual method (of LP) [9].

3. Disclosable Loop

The classical approach to the problem of synthesis of optimal systems is based on deterministic models and does not use any information about perturbations. The introduction of non-deterministic models makes it possible to estimate the quality of the control process regarding disturbances.

To begin with, we analyze the simplest problem of optimal control of a dynamic object under conditions of multiple uncertainty:

$$c'x(t^*) \to \max; \quad \dot{x}(t) = A(t)x(t) + b(t)u(t) + m(t)w(t), \tag{3.1}$$
$$x(t^*) \in X^*, \quad u(t) \in U, \quad w(t) \in W, \quad t \in T.$$

Problem (3.1) differs from problem (2.1) by the presence of an unknown function $w(\cdot)$, which can be realized as any sectionally continuous function with values from the compact set W.

The control action $u(\cdot)$ and the perturbation $w(\cdot)$ in problem (3.1) generate the trajectory $x\big(t|t_*, x_0, u(\cdot), w(\cdot)\big)$, $t \in T$.

Definition 3.1. The set of possible terminal states of the control object (3.1), corresponding to the available control $u(\cdot)$ $\mathscr{X}\left(t^*, u(\cdot)\right) = \{x \in \mathbb{R}^n : x = x\big(t^*|t_*, x_0, u(\cdot), w(\cdot)\big),\ w(t) \in W,\ t \in T\}$, is called a distribution of the terminal state at the moment t^*.

Definition 3.2. An available control action $u_g(\cdot)$ is called a guaranteeing program for problem (3.1) if $\mathscr{X}\left(t^*, u_g(\cdot)\right) \subset X^*$.

Let the quality of the guaranteeing program be estimated by the number

$$J\big(u_g(\cdot)\big) = \min c'x\big(t^*|t_*, x_0, u_g(\cdot), w(\cdot)\big),\ w(t) \in W,\ t \in T$$

(guaranteed value of the quality index).

The optimal guaranteeing program $u_g^0(\cdot)$ is defined by the equality

$$J\big(u_g^0(\cdot)\big) = \max J\big(u_g(\cdot)\big),\ u_g(t) \in U,\ t \in T.$$

The optimal program $u_g^0(\cdot)$ transfers system (3.1) to the terminal set X^* at time t^* with a guarantee and provides the maximum guaranteed value of the quality index $J_g\big(u(\cdot)\big)$.

To determine the guaranteeing positional solution of problem (3.1), consider the set of problems:

$$c'x(t^*) \to \max, \quad \dot{x}(t) = A(t)x(t) + b(t)u(t) + m(t)w(t), \tag{3.2}$$
$$x(\tau) = z, \quad x(t^*) \in X^*, \quad u(t) \in U, \quad w(t) \in W, \quad t \in T(\tau).$$

Let $u_g^0(t|\tau, z)$, $t \in T(\tau)$ be the optimal guaranteed program for problem (3.2) for position (τ, z), and X_τ be the set of all states $z \in \mathbb{R}^n$ for which there are guaranteed programs for the problem (3.1).

Definition 3.3. The function $u_g^0(\tau,z) = u_g^0(\tau|\tau,z)$, $z \in X_\tau$, $\tau \in T_h$, is called an extremal discrete open loop (a positional solution of problem (3.1)).

The problem of constructing an extreme discrete feedback, as well as the problem (2.1) in the class of discrete control actions is reduced to the linear programming problem and solved by a dual method [3, 9, 10].

4. Closable Loop

When determining the closable loop, the object was limited at each current moment of time $\tau \in T_h$, but the information that the closures can also take place at future times $t > \tau$ was not used. When using deterministic models, this circumstance does not play any role. In the case of non-deterministic models, the situation changes and additional information significantly affects the control process [12, 13].

Below are some results obtained for the case of a single closure. Let the trajectory of the control object be described by the system of equations

$$\dot{x}^*(t) = A(t)x^*(t) + b(t)u(t) + m(t)w^*(t), \quad x^*(t_*) = x_0, \tag{4.1}$$

the trajectory of the model by the system

$$\dot{x}(t) = A(t)x(t) + b(t)u(t) + m(t)w(t), \quad x(t_*) = x_0, \tag{4.2}$$

where $w^*(t)$, $w(t)$, and $t \in T$ are perturbations acting on the object and on the model (accordingly).

Let t_1 be the moment of time $t_* < t_1 < t^*$ at which the state of the object becomes known, i.e., the object at this moment will be closed. Every triple $\{u(\cdot), w^*(\cdot), w(\cdot)\}$ generates unique trajectories of (4.1) - (4.2). The function $x^*(t)$, $t \in T$ is a continuous solution to system (4.1), the function $x(t)$, $t \in T$ is a piecewise continuous solution to the system (4.2) with a point of discontinuity at the moment of closure: $x(t_1 + 0) = x^*(t_1)$.

The terminal state of system (4.2) has the form:

$$x(t^*) = F(t^*,t_*)x_0 + \int_{t_*}^{t^*} F(t^*,t)b(t)u(t)dt + \int_{t_*}^{t_1} F(t^*,t)m(t)w^*(t)dt + \int_{t_1}^{t^*} F(t^*,t)m(t)w(t)dt,$$

where $F(t,\tau)$, $t \geq \tau$, $t,\tau \in T$ is the fundamental matrix of solutions of the homogeneous part of system (4.2), $F(t,\tau) = F(t)F^{-1}(\tau)$, $F(t) \in \mathbb{R}^{n\times n}$, $t \in T$, and

$$\dot{F}(t) = A(t)F(t), \quad F(t_*) = E$$

(E — identity matrix).

Further, following Section 2, we obtain programmed and positional solutions for control of the object in real time on the principle of one-time closed loop. Similarly, the problem of extreme control in real time is solved on the principle of repeatedly closable loop.

Let us find the relations that describe the guaranteeing program (4.1). According to the definition, the available control action $u(t) \in U$ is a guaranteeing program only when inequalities are met for all possible perturbations $w(t) \in W$, $t \in T$:

$$g_{*i} \leq h_i' x(t^*) \leq g_i^*, \; i \in I. \tag{4.3}$$

On the available control action $u(\cdot)$, the restriction (4.3) will be carried out for all $w(t) \in W$ if and only if

$$\max_{w(t)\in W, t\in T} h_i' x(t^*) \le g_i^*, \qquad \min_{w(t)\in W, t\in T} h_i' x(t^*) \ge g_{*i},\ i\in I.$$

Applying formula (4.3), we obtain

$$h_i'F(t^*,t_*)x_0 + \int_{t_*}^{t^*} h_i'F(t^*,s)b(s)u(s)ds + \max_{w(t)\in W, t\in T} \int_{t_*}^{t^*} h_i'F(t^*,s)m(s)w(s)ds \le g_i^*,\ i\in I \quad (4.4)$$

and

$$h_i'F(t^*,t_*)x_0 + \int_{t_*}^{t^*} h_i'F(t^*,s)b(s)u(s)ds + \min_{w(t)\in W, t\in T} \int_{t_*}^{t^*} h_i'F(t^*,s)m(s)w(s)ds \ge g_{*i},\ i\in I. \quad (4.5)$$

From (4.4) and (4.5), it follows that

$$\int_{t_*}^{t^*} h_i'F(t^*,s)b(s)u(s)ds \ge \tilde g_{*i}, \quad \int_{t_*}^{t^*} h_i'F(t^*,s)b(s)u(s)ds \le \tilde g_i^*,\ i\in I,$$

where

$$\tilde g_i^* = g_i^* - \gamma_i^* - h_i'F(t^*,t_*)x_0,$$
$$\tilde g_{*i} = g_{*i} - \gamma_{*i} - h_i'F(t^*,t_*)x_0, i\in I,$$
$$\gamma_{*i} = \min_{w(t)\in W, t\in T} \int_{t_*}^{t^*} h_i'F(t^*,s)m(s)w(s)ds,\ i\in I,$$

and

$$\gamma_i^* = \max_{w(t)\in W, t\in T} \int_{t_*}^{t^*} h_i'F(t^*,s)m(s)w(s)ds,\ i\in I.$$

Thus the available control action $u(t)\in U$, $t\in T$ is a guaranteeing program if and only if the inequalities are fulfill

$$\tilde g_{*i} \le \int_{t_*}^{t^*} h_i'F(t^*,s)b(s)u(s)ds \le \tilde g_i^*,\ i\in I. \quad (4.6)$$

Let us calculate the values of the quality indicator (4.5) and the guaranteed value of the quality index (4.6) on the available control action $u(t)\in U$, $t\in T$:

$$J\big(u(\cdot)\big) = c'F(t^*,t_*)x_0 + \int_{t_*}^{t^*} c'F(t^*,s)b(s)u(s)ds + \min_{w(t)\in W, t\in T} \int_{t_*}^{t^*} c'F(t^*,s)m(s)w(s)ds, \quad i\in I.$$

$$J\big(u^0(\cdot)\big) = c'F(t^*,t_*)x_0 + \max_{u(t)\in U, t\in T} \int_{t_*}^{t^*} c'F(t^*,s)b(s)u(s)ds +$$
$$+ \min_{w(t)\in W, t\in T} \int_{t_*}^{t^*} c'F(t^*,s)m(s)w(s)ds,\ i\in I.$$

Accordingly, the construction of the optimal guaranteeing program of the problem (4.1), (4.2) is reduced to solving three problems:

$$\int_{t_*}^{t^*} h_i'F(t^*,s)m(s)w(s)ds \to \min_{w(t)\in W, t\in T}, \quad \int_{t_*}^{t^*} h_i'F(t^*,s)m(s)w(s)ds \to \max_{w(t)\in W, t\in T}, \quad i\in I,$$

$$\int_{t_*}^{t^*} c'F(t^*,s)b(s)u(s)ds \to \max_{u(t)\in U, t\in T}, \quad (4.7)$$

$$\tilde{g}_{*i} \leq \int_{t_*}^{t^*} h_i' F(t^*, s) b(s) u(s) ds \leq \tilde{g}_i^*, \; i \in I.$$

In a particular case, when the restrictions on the control and disturbing actions $u(\cdot),\ w(\cdot)$ are of the form:

$$|u(t)| \leq L_u, \quad |w(t)| \leq L_w, \quad t \in T_h,$$

we have

$$\gamma_{*i} = \min_{w(t) \in W, t \in T} \int_{t_*}^{t^*} h_i' F(t^*, s) m(s) w(s) ds = -L_w \int_{t_*}^{t^*} \left| h_i' F(t^*, s) m(s) \right| ds, \; i \in I,$$

and

$$\gamma_i^* = \max_{w(t) \in W, t \in T} \int_{t_*}^{t^*} h_i' F(t^*, s) m(s) w(s) ds = L_w \int_{t_*}^{t^*} \left| h_i' F(t^*, s) m(s) \right| ds,, \; i \in I.$$

To construct an optimal guarantee program, we obtain the problems

$$\int_{t_*}^{t^*} h_i' F(t^*, s) m(s) w(s) ds \to \min, \; \int_{t_*}^{t^*} h_i' F(t^*, s) m(s) w(s) ds \to \max, \; |w(t)| \leq L_w, \; t \in T, \; i \in I,$$

$$\int_{t_*}^{t^*} c' F(t^*, s) b(s) u(s) ds \to \max, \quad |u(t)| \leq L_u, \; t \in T, \; i \in I, \tag{4.8}$$

$$\begin{aligned} & g_{*i} + L_w \int_{t_*}^{t^*} \left| h_i' F(t^*, s) m(s) \right| ds - h_i' F(t^*, t_*) x_0 \\ & \leq \int_{t_*}^{t^*} h_i' F(t^*, s) b(s) u(s) ds \\ & \leq g_i^* - L_w \int_{t_*}^{t^*} \left| h_i' F(t^*, s) m(s) \right| ds - h_i' F(t^*, t_*) x_0, \; i \in I. \end{aligned}$$

The methods for solving the above optimal control problems were given in [8, 14]. To introduce the concept of a positional solution $u^0(t|\tau, z)$, $\tau \in T$, $z \in \mathbb{R}^n$ to (4.1) and (4.2), consider a set of problems:

$$c' x(t^*) \to \max; \quad \dot{x}(t) = A(t) x(t) + b(t) u(t) + m(t) w(t), \quad x(\tau) = z, \tag{4.9}$$

$$x(t^*) \in X^*, \quad u(t) \in U, \quad w(t) \in W, \; t \in T(\tau),$$

depending on n-vector $z \in \mathbb{R}^n$ and scalar $\tau \in T$. Let $u^0(t|\tau, z)$, $\tau \in T(\tau)$ is the optimal guaranteeing program for a position (τ, z), $\tau \in T_h$, $z \in \mathbb{R}^n$, X_τ — the set of all states for which exist guaranteeing programs for problem (4.8).

Suppose that $u^0(t|\tau, z)$, $t \in T(\tau)$ is the optimal guaranteeing program of problem (4.7) for the position (τ, z); $\tau \in T_h$, $z \in \mathbb{R}^n$, X_τ — the set of all states $z \in \mathbb{R}^n$ for which there are guaranteeing programs of the problem (4.7).

Definition 4.1. The function

$$u^0(\tau, z) = u^0(\tau|\tau, z), \; z \in X_\tau, \; \tau \in T \tag{4.10}$$

is called the optimal guaranteeing program for the position (τ, z).

Definition 4.2. The function (4.10) defined on all possible positions (τ, z), $z \in X_\tau$, $\tau \in T$, is called the optimal guaranteeing open loop control (positional solution to problem (4.1)).

Let us write down the relations for constructing the guaranteeing program of the problem (4.1), (4.2):

$$\int_{t_*}^{t^*} h_i' F(t^*,s)m(s)w(s)ds \to \min, \quad \int_{t_*}^{t^*} h_i' F(t^*,s)m(s)w(s)ds \to \max, \quad w(t) \in W,\ t \in T,\ i \in I,$$

$$\int_{t_*}^{t^*} c' F(t^*,s)b(s)u(s)ds \to \max, \quad u(t) \in U,\ t \in T,\ i \in I,$$

$$\tilde{g}_{*i} \le \int_{t_*}^{t^*} h_i' F(t^*,s)b(s)u(s)ds \le \tilde{g}_i^*,\ i \in I,$$

where $\tilde{g}_i^* = g_i^* - \gamma_i^*(t_*) - h_i' F(t^*,t_*)z$, $\tilde{g}_{*i} = g_{*i} - \gamma_{*i}(t_*) - h_i' F(t^*,t_*)z$, $i \in I$, $t_* \in T_h$, $z \in X_\tau$; and

$$\gamma_{*i}(t_*) = \min_{w(t)\in W, t\in T} \int_{t_*}^{t^*} h_i' F(t^*,s)m(s)w(s)ds,\ \gamma_i^*(t_*) = \max_{w(t)\in W, t\in T} \int_{t_*}^{t^*} h_i' F(t^*,s)m(s)w(s)ds. \tag{4.11}$$

Calculation of the current value of the positional solution $u^0(t|\tau,z)$, $t \in T$ of the problem (4.1), (4.2) for the position (τ,z) comes to the procedure of correction of program solutions.

5. The Optimal Real-time Control Problem with a Guarantee

Supplement the formulation of problem (4.1) with the following condition. Let t_1 be such a moment of time $t_* < t_1 < t^*$ that the state $x(t_1)$ of the object (4.1) generated by the available control action $u(t) \in U$, $t \in T$ can be accurately measured at any realization of a possible perturbation $w(t) \in W$, $t \in T$. From now on, let t_1 be the moment of closure. And we compare the state $x(t_1)$ of the object with the state of mathematical object $z(t_1)$ [4]:

$$c'z(t) \to \max,\ \dot{z}(t) = A(t)z(t) + b(t)u(t),\ z(t_*) = x_0,\ u(t) \in U,\ z(t^*) \in X^*,\ t \in T. \tag{5.1}$$

Thus

$$x(t_1) - z(t_1) = \int_{t_*}^{t_1} F(t_1,s)m(s)w(s)ds.$$

From [10], for the states of the object (4.9) and the mathematical model (5.1) to coincide at the moment t_1 $\big(x(t_1) = z(t_1)\big)$, it is necessary and sufficient that the following equality be realized

$$\int_{t_*}^{t_1} F(t_1,s)m(s)w(s)ds = 0,\ w(t) \in W,\ t \in T. \tag{5.2}$$

Equality (5.2) is interpreted as an additional restriction on the set of possible perturbations $w(\cdot)$, which corresponds to new information. Therefore, when calculating the extreme possible perturbations (4.11) of (4.1) and (4.2), we add the condition to the initial constraints (5.2).

The guaranteeing program of (4.1) and (4.2) with one moment of closure t_1 is reduced to solving the following problems:

The problem with unknown perturbation action $w(\cdot)$:

$$\int_{t_*}^{t^*} h_i' F(t^*,s)m(s)w(s)ds \to \min_{w(t)\in W, t\in T}, \quad \int_{t*}^{t^*} h_i' F(t^*,s)m(s)w(s)ds \to \max_{w(t)\in W, t\in T},\ i \in I, \tag{5.3}$$

$$\int_{t_*}^{t^*} F(t_1,s)m(s)w(s)ds = 0,\ w(t) \in W,\ t \in T.$$

The problem with unknown available control actions:

$$\int_{t_*}^{t^*} c'F(t^*,s)b(s)u(s)ds \to \max_{u(t)\in U, t\in T}, \ i\in I,$$

$$u(t)\in U, t\in T, \quad \tilde{g}_{*i} \leq \int_{t_*}^{t^*} h_i' F(t^*,s)b(s)u(s)ds \leq \tilde{g}_i^*, \ i\in I.$$

When dealing with the positional solution of (4.1) and (4.2), each current value of the control action is obtained from the value of the previous guaranteeing program and is found by the dual linear programming method using the procedure correction of the optimal support; see, e.g., [4, 11].

Since the restrictions of problems (5.3) are obtained by adding conditions (5.2) to constraints (4.2), the optimal value of the quality index $J\left(u^0(\cdot)\right)$ satisfies the inequality $J\left(u^0(\cdot)\right) \leq J_1\left(u^0(\cdot)\right)$, where $J_1\left(u^0(\cdot)\right)$ complies with the quality index of the object with constraints (4.2) and (5.2). The set of possible perturbations $w(\cdot)$ in the case of a single closure narrows down to a set

$$\left\{ w(\cdot): \int_{t_*}^{t_1} F(t_1,s)m(s)w(s)ds = 0 \right\}.$$

The problem of guaranteeing control for linear time-varying systems with the use of multiple time closures $T_3 = \{t_j \in T_h, \ j=\overline{1,p}\}$, $t_* < t_1 < t_2 < \ldots < t_p < t^*$ is studied by a similar way.

6. Conclusion

This paper offers algorithm's solutions to construct real-time guaranteeing control actions for a linear dynamical system under conditions of constantly acting limited perturbation actions. The method is based on the use of discrete control actions, reduction of a non-deterministic problem to a linear programming problem, and a procedure for correcting the current values of guaranteeing control actions.

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