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SOLVING RESOURCE ALLOCATION PROBLEMS IN DISASTER MANAGEMENT USING LINEAR PROGRAMMING RELAXATIONS

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Dedication to the memory of Professor Hoang Tuy

Abstract. In this paper, we propose a new efficient approach for solving some special types of integer linear programming problems, frequently appearing in practical applications. Such problems require an integer solution and therefore, in most cases, the computations rely on integer and mixed-integer linear programming solvers. In general, these solvers can not handle large scaled problems. To avoid this difficulty, many researchers applied some heuristic-based approaches or introduced a clustering structure to reduce the size of the problem. We demonstrate that there exists a large class of practical applications, in particular, disaster management and resource location-allocation problems whose exact solutions can be obtained by applying the simplex method (linear programming) without relying on integer solvers. This approach has several advantages. First of all, any modern optimisation solvers can handle large-scaled linear programming problems. Second, the optimal solutions are the solutions to the original problems without any artificial constructions.

Keywords. Integer programming; Optimisation; Totally unimodular matrices; k-medoid clustering; Resource allocation.

1. Introduction

A large class of applications can be formulated as integer or mixed-integer linear programs. In these problems, all or some of the variables are integers. This makes the corresponding optimisation problems much harder to solve. Moreover, in some cases, we can not even guarantee that an obtained solution is optimal. Many operations research practitioners, knowing that integer linear programming problems are NP-complete, tend to remodel the original problems through introducing artificial clustering and certain hierarchical structures. This approach reduces the size the original problem leading to solving several smaller size problems [2, 3, 9]. While this approach is very reasonable for a general type of integer linear problems, it destroys

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some important characteristics of the original problem. Our approach preserves these characteristics for further exploration and exploitation, since it can be efficiently handled without any additional (artificial) constructions.

In this paper, we identify a rich class of integer linear problems whose optimal solution can be obtained by applying the simplex method [8], a powerful methods developed for classical linear programs. We demonstrate that in these problems the feasible set is a polytope, whose vertices are integers and therefore the simplex method finds an optimal solution which is also integer.

There are several advantages of our approach. First of all, we can guarantee that the obtained solution is integer and optimal. Second, linear programming techniques can deal with very large problems, while the applicability of integer and mixed-integer techniques to same size problems may be problematic. Third, there is no need to introduce any clustering-based approximation to the original problem.

Many disaster management and resource location-allocation approaches are based on optimisation techniques. In particular, the problems are formulated as mathematical programs and then solved using available optimisation techniques [30].

In this paper we consider three applications taken from the field of disaster management. In the first two applications, the constraint matrices are totally unimodular and therefore the application of the classical simplex method leads to an optimal solution. The third application is based on k-medoid clustering [15] and therefore it is also applicable to clustering problems. k-medoid problems are known to be NP-complete [22] and hence the application of linear programming techniques does not necessary reach an optimal solution in a single step, but the interpretation of branching in this particular problem is the first step in better understanding of the problem. The list of applications where our approach is efficient goes well beyond the areas studied in this paper.

The paper is organised as follows. In Section 2, we provide known results from linear programming relevant to this study. In Section 3, we give an overview of resource location-allocation, disaster management, and k-medoid clustering problems. In Section 4, we present the results of the numerical experiments. Finally, in Section 5, we provide our conclusions and discussions.

2. THE SIMPLEX METHOD AND LINEAR PROGRAMMING

2.1. **Linear Programming and transportation problem.** There are many efficient methods for solving linear programming problems (LPPs). One of them is the simplex method, originally developed by Dantzig in 1947 [8]. It has been demonstrated in 1972 by [16] that the worst-case complexity of the simplex method, as formulated by Dantzig, is exponential time. Despite this result, the simplex method is remarkably efficient and included in most linear programming packages. The simplex method starts at a feasible vertex of the constraint polytope. Then it moves to an adjacent vertex, where the objective function value is at least as good as it is at the original vertex. Since the number of vertices is finite, this method terminates in a finite number of steps. If all the vertices of the constraint polytope have integer coordinates, then an optimal solution (can be more than one) is integer.

Transportation problems form a special class of LPPs. In most operations research textbooks, this problem is formulated in a way related to transportation. It is possible, however, to apply the same approach to other types of problems, including location analysis and allocation problems. An excellent overview of such problems and also more advanced models can be found in [9, 18].

Consider an example of transportation problem formulation. Goods are produced at m factories (also called sources) $S_1,\dots,S_m$ and sold at n markets (also called destinations): $D_1,\dots,D_n$. The supply available at source S_i is $s_i \geq 0$ units, the demand at destination D_j is $d_j \geq 0$ units and the transportation cost of one unit from S_i to D_j is $c_{ij} \geq 0$. We have to identify which sources should supply which destinations to minimise total transportation costs. Let x_{ij} be the number of units to be sent from S_i to D_j. Then the corresponding optimisation problem can be formulated as follows:

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leq s_i,\ i=1,\dots,m; \sum_{i=1}^{m} x_{ij} \geq d_j,\ j=1,\dots,n;$$

$$x_{ij} \geq 0,\ i,j=1,\dots,n.$$

It is well known that if all supplies and demands are integers, then all the vertices of the feasible set are integers and therefore there exists an optimal solution x_{ij}, which is integer. Hence, the simplex method applied to a transportation problem, terminates at an optimal solution that is also integer.

2.2. **Integer problems with totally unimodular constraint matrices.** A comprehensive review on integer programming can be found in [25]. Theorem 19.3 of this book (p. 268) covers the conditions when the vertices of the feasible sets are integers and therefore an optimal solution found at a vertex is integer. In this paper, we only use conditions (i), (iii), and (iv) of the theorem (originally proved in [14] and [12]). A simplified version of this theorem, formulated for our study, is as follows.

Theorem 2.1. *Let A be a matrix with entries $\{0,1,-1\}$. Then the following are equivalent:*

(1) matrix A is totally unimodular, that is, each square submatrix of A has determinant 0, 1, or -1;
(2) for all integral vectors, a, b, c, and d, the polytope $\{x|c \leq x \leq d,\ a \leq Ax \leq b\}$ has only integral vertices;
(3) each collection of the columns of A can be split into two parts so that the sum of the columns in one part minus the sum of the columns in the other part is a vector with entries $\{0,1,-1\}$;

The class of totally unimodular matrices is closed under a number of operations, including transposition and multiplication a row (column) by -1. Matrix A is totally unimodular if and only if matrix $[IA]$ (where I is an identity matrix of the corresponding dimension) is totally unimodular.

In the next section, we formulate three types of integer problems appearing in disaster management. We demonstrate that in the first two models the corresponding system matrices are totally unimodular and therefore their optimal solutions (reached at vertices) are integers and hence can be obtained by applying the simplex method. The third application is not so straightforward, but still benefits from preserving the original formulation rather than producing clustering based approximations.

3. Mathematical Modelling in Disaster Management

3.1. **Problem overview.** In emergency relief operations, resource allocation problems (RAPs) are often a complex challenge due to a number of issues, such as dealing with crucial demands, time restrictions, competing priorities, availability, uncertainties and other constraints [27]. Emergency resources can be grouped into non-expendable and expendable resources [3, 26]. Non-expendable resources are non-consumable and renewable: emergency personnel, volunteers, etc. Expendable resources are consumable and cannot be renewed in the emergency: medical supplies, water, food and fuel, etc. Failure to assign adequate resources in a timely manner has been the main cause of adverse impacts in disaster situations [20, 21, 24].

RAPs in disaster management are usually formulated as integer linear programming problems (ILPPs) or mixed-integer linear programming problems (MILPPs) [3, 5, 11]. Most of the methodologies in natural disaster response phase focus on developing an MILPP or ILPP models just for one type of the resources. A literature review reveals that models with different kinds of resources in disaster management have been discussed quite rarely [3, 19, 20].

Furthermore, finding the location of relief centres (e.g. medical centres, distribution centres) are vital in disaster management [4]. This problem can be addressed as a facility location problem [1] or a clustering problem [2, 23]. To find a solution for the location of relief centres, an MILP model is formulated and solved for small cases, while larger cases require several heuristic approaches. In most cases, heuristic algorithms produce non-optimal solutions [4].

3.2. **Reduction to classical linear programming.**

3.2.1. *Expendable resources.* The problem of allocating expendable resources can be formulated as a transportation problems. It is enough to think about incident points as "Markets'' (each market demand corresponds to an incident point demand), while the relief centres are "Factories'' (each factory capacity corresponds to a processing centre capacity). The transportation costs are "processing and transportation time''.

A mathematical formulation for the case of expendable resources is as follows

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{i=1}^{m} x_{ij} \geq d_j, \ j = 1, \dots, n;$$

$$\sum_{j=1}^{n} x_{ij} \leq s_i, \ i = 1, \dots, m;$$

$$x_{ij} \geq 0, \ i = 1, \dots, n, \ j = 1, \dots, m,$$

$$x_{ij} \text{ are integers}, \ i = 1, \dots, m, \ j = 1, \dots, n,$$

where $d_i, \ i = 1, \dots, m$ are incident point demands and $s_j, \ j = 1, \dots, n$ are relief centre capacities.

The feasible set of this problem is as follows (without sign constraints and integer requirement):

$$\begin{bmatrix} -I_n & -I_n & -I_n & \dots & -I_n \\ e_n & 0_n & 0_n & \dots & 0_n \\ 0_n & e_n & 0_n & \dots & 0_n \\ 0_n & 0_n & e_n & \dots & 0_n \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0_n & 0_n & 0_n & \dots & e_n \end{bmatrix} X \leq b, \tag{3.1}$$

where $b \in \mathbb{R}^{(n+m)}$ represents the corresponding demands and supplies and therefore b is integral;
$X \in \mathbb{R}^{mn}$ is the vector of decision variables; $I_n$is an identity matrix of size m;
$e_n = (1, 1, \dots, 1)$ is a row vector, whose components are all equal to 1: $e_n^T \in \mathbb{R}^n$;
$0_n = (0, 0, \dots, 0)$ is a row vector, whose components are all equal to 0: $0_n^T \in \mathbb{R}^n$;
the system matrix $A \in \mathbb{R}^{(n+m)\times(mn)}$.

Theorem 3.1. *The system matrix A from (3.1) is totally unimodular.*

Proof. Consider matrix B obtained from A^T by multiplying the first n columns of A^T by -1. Matrix A is totally unimodular if and only if matrix B is totally unimodular.

Assign the first n columns of B to part I and the remaining columns to part II, and assume that one or more columns may be removed from the total collection of columns. The sum of the columns in part I is an (mn)-dimensional vector S_1 whose components are 0 or 1. The sum of the columns in part I is an (mn)-dimensional vector S_2 whose components are 0 or 1. Therefore the components of $S_1 - S_2$ are 0, 1, or -1. Hence, by Theorem 2.1, we conclude that matrices B and A are totally unimodular, and all the vertices of the feasible set have integer coordinates. □

We would like to underline the following important conclusions.

(1) The problem appears to be an IPP, however, this problem can be reduced to a well-known and extensively studied classical Transportation problem.
(2) The constraint matrix of this problem is totally unimodular and therefore one can also conclude that an optimal integer solution to this problem can be found by applying the simplex method.
(3) One can reduce the solution of this problem to solving a linear programming problem whose optimal solution set contains an integer solution. This solution can be found by applying the simplex method to the corresponding linear relaxation and we can guarantee that the obtained optimal solution is integer.

3.2.2. *Non-expendable resources.* In the case of non-expendable resources, the problem can also be formulated as an ILPP, where some of the summation constraints from a classical transportation problem are replaced with maximisation. This problem is not a classical transportation problem, but its relaxation (ignoring integer constraints) can be formulated as an LPP. It can be demonstrated that the application of the simplex method leads to an integer optimal solution.

A mathematical formulation for the case of non-expendable resources is as follows

$$\min \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{i=1}^{m} x_{ij} \geq d_j, \; j = 1, \dots, n;$$
$$\max_{j=1,\dots,n} x_{ij} \leq s_i, \; i = 1, \dots, m; \tag{3.2}$$
$$x_{ij} \geq 0, \; i = 1, \dots, n, \; j = 1, \dots, m,$$
$$x_{ij} \text{ are integers}, \; i = 1, \dots, m, \; j = 1, \dots, n, \tag{3.3}$$

where $d_i, \; i = 1, \dots, m$ are incident point demands and $s_j, \; j = 1, \dots, n$ are relief centre capacities. A relaxation of this problem, obtained by removing the last constraint (3.3), can be formulated as an LPP by replacing constraints (3.2) with equivalent systems of linear inequalities:

$$x_{ij} \leq s_j, \; j = 1, \dots, n, \; i = 1, \dots, m.$$

The feasible set of this problem can be formulated as follows (without sign constraints and integer requirement):

$$\begin{bmatrix} -I_n & -I_n & -I_n & \dots & -I_n \\ & & I_{mn} & & \end{bmatrix} X \leq b, \tag{3.4}$$

where $b \in \mathbb{R}^{n(m+1)}$ represents the corresponding demands and supplies and therefore the components of b are integers; $X \in \mathbb{R}^{mn}$ is the vector of decision variables; I_n is an identity matrix of size n; I_{mn} is an identity matrix of size mn; the system matrix $A \in \mathbb{R}^{n(m+1)\times(mn)}$.

Theorem 3.2. *The system matrix A from (3.4) is totally unimodular.*

Proof. A is totally unimodular if and only if matrix

$$B = \begin{bmatrix} I_n & I_n & I_n & \dots & I_n \end{bmatrix}$$

is totally unimodular. Matrix B is totally unimodular if and only if I_n is totally unimodular. Indeed, one can assign any collection of columns of I_n to part I and the remaining columns to part II. The difference of the corresponding columns sums contains 1 and -1 as the components. □

Therefore, similar to the case of expendable resources, we can reduce the corresponding ILPP to an LPP whose vertices are integers.

3.2.3. *Relief centre location and k-medoid clustering.* In this application, the distance matrix between all the incident points is given. The goal is to select k points in such a way that, after assigning all the remaining points to the nearest selected point (cluster centre), the total sum of distances between the points and centres is minimal. Each cluster centre is a relief centre, whose optimal location (selection among the incident points) is the objective. In this application, we assume that the demand of the incident points can be covered regardless of the allocation, since the main objective is to minimise the total distance. This kind of clustering is called k-medoid.

This problem is known to be NP-complete [22] and therefore, in general, it is not reasonable to expect the constraint matrix to be totally unimodular. Therefore, the relaxation of the problem to a linear programming problem may provide a non-integer optimal solution.

There exist several methods for k-medoid. One of the most efficient is Partitioning Around Medoids (PAM) algorithm [15]. Most such methods are based on iterative clustering rather than optimisation and employ a number of heuristics [18]. Moreover, they rely on a particular norm. For example, PAM developed in [15] relies on L_1.

This problem can be formulated as an ILPP [29]. Assume that there are n demand points in total and the distance matrix

$$\mathbf{D} = \{d_{ij}\},\ i = 1,\dots,n,\ j = 1,\dots,n.$$

The goal is to select k points as relief centres. The decision variables are binary: $x_{ij} \in \{0,1\},\ i = 1,\dots,n,\ j = 1,\dots,n$ and $y_i \in \{0,1\},\ i = 1,\dots,n$. Variable y_i is 1 if incident point i is treated as a relief centre, otherwise, this variable is zero. Variable x_{ij} is 1 if incident point i was assigned to point j (zero otherwise). The corresponding optimisation problem is as follows:

$$\min \sum_{i=1}^{n} \sum_{j=1}^{n} d_{ij} x_{ij} \tag{3.5}$$

subject to

$$\sum_{i=1}^{n} x_{ij} = 1,\ j = 1,\dots,n; \tag{3.6}$$

$$x_{ij} \leq y_i,\ i,j = 1,\dots,n; \tag{3.7}$$

$$\sum_{i=1}^{n} y_i = k; \tag{3.8}$$

$$x_{ij},\ y_i \in \{0,1\},\ i,j = 1,\dots,n. \tag{3.9}$$

Constraints (3.6) ensure that each incident point is assigned to a single relief centre. Constraints (3.7) ensure that an incident point i can only be assigned to an incident point j if this point is also a relief centre. Finally, constraint (3.8) ensures that exactly k points are selected as relief centres. It is clear that problem (3.5)-(3.9) is an ILPP.

While we can not guarantee that all the vertices represent integer solutions, it is clear that every feasible integer solution is a vertex of the relaxed convex feasible set, where constraint (3.9) is replaced by $x_{ij},\ y_i \in [0,1],\ i,j = 1,\dots,n$. Indeed, since any integer feasible solution contains only "zero" or "one", these vector can not be obtained as a convex combination of two other vectors, unless one of these vectors is exactly the solution. This observation is due to the fact that we are working on a unit hypercube, whose vertices can only have "zero" or "one" as their components.

There are two main approaches to ILPPs. One of them is the branch and bound method, originally developed in [17], and another one is Gomory cuts [13]. Most experts, including Gomory himself, considered the Gomory cuts to be impractical and ineffective. This method was significantly improved by Gérard Cornuéjols and coauthors [7].

The application of branching to any non-integer component results into two possibility: to force this component to be exactly "one" or "zero" and move to another vertex. So, it is beneficial to start branching at a vertex obtained by applying the simplex method to the corresponding linear relaxations. Moreover, in many cases the application of the simplex method leads to an integer solution.

Another interesting issue here is that the constraint polytope does not depend on the geometry of the problem (location of incident points). Indeed, if n (number of incident points) and k (number of centres) are fixed, the constraint polytope remains the same and therefore can be reused for another (updated) problem. Similar approach has been proposed in [28] where it was beneficial to apply a non-standard approach to solve convex problems with slightly updated matrices. In the current application, it is beneficial to find an accurate approximation of the constraint set, whose vertices are integers. This can be done by construction, for example, the Gomory-Chavátal Closure [10]. We are planning to study this very promising method in the future.

4. Numerical Experiments

To demonstrate the applicability of these RLP models, a hypothetical disaster relief problems have been solved using CPLEX Solver 12.7 on a 3.4 GHz processor with 4 GB of RAM. The incident points and hospital locations were randomly generated using uniform distribution.

4.1. **Expendable and non-expendable resources.** The results obtained for expendable and non-expendable resource allocation were comparable for both implementations: through the simplex method (LPP) and branch and bound (ILPP). Namely, the computational time and the optimal value of the cost function were almost the same. This confirms that the branch and bound method terminates after just very few branching stages.

4.2. **Relieve centre location via *k*-medoid.** In the case of the relief centre location experiments, the situation is different, since the corresponding optimisation problems are more complex. Table 1 contains the results of the numerical experiments. In this table we compare the computational time for the simplex method and IP implementations (branch and bound), the optimal cost value is the same for both implementations and therefore only one column is provided. k represents the number of clusters. These results indicate that the computational time is significantly lower for the simplex method.

The computational time difference may be attributed to the fact that the branch and bound method relies on interior-point methods and therefore it does not always terminate at a vertex before first branching. On the top of this, the simplex method reached integer solutions in all the experiments. All these confirm that it is very efficient to use the classical simplex method in k-medoid models.

Note that in some cases we can have a non-integer solution. In this case, this vertex can be excluded using, for example, Gomory-Chvátal Closure [6, 10, 13], which can be constructed in polynomials time [10].

Overall, the results can be summarised as follows.

- Our numerical experiments empirically confirm that the direct application of the simplex method often leads to an integer solution.
- From purely computational time estimation, the branch and bound method terminates after 1-3 branching iterations.
- The recommendation is to start with the simplex method and apply Gomory-Chvátal Closure approach if the solution is not integer.

5. Discussion and Future Research Directions

In this paper, we demonstrated that there are a number of practical applications formulated as ILPPs, whose solution can be obtained by applying the simplex method or branch and bound with very few branching. This important observation demonstrated that before applying an integer solver one needs to explore the constraint set and avoid any artificial constructions that may potentially destroy the totally modular constraint matrices.

TABLE 1. Relief centre allocation experiments

Number of data points	k	Cost function value	Time (simplex)	Time (IP, branch and bound)
20	10	512.3478602	0.297	0.562
20	5	311.3343592	0.39	0.391
40	10	679.4089905	0.063	0.078
40	5	425.939219	0.063	0.067
60	10	848.5919719	0.14	0.235
60	5	490.5917792	0.187	0.191
80	10	961.283263	0.235	0.359
80	5	565.3424002	0.25	0.351
100	10	1109.977437	0.406	0.594
100	5	640.0424524	0.438	0.532
120	10	1173.702679	0.594	0.797
120	5	676.7603006	0.64	0.718
140	10	1241.879626	0.765	1.078
140	5	715.5979726	0.86	1
160	10	1297.640491	1.016	1.453
160	5	770.3419758	1.109	1.36
180	10	1346.426155	1.328	1.828
180	5	801.1360661	1.219	1.765
200	10	1425.259571	1.687	2.344
200	5	860.7051955	1.453	2.438
400	10	1954.177312	8.844	11.672
400	5	1170.263786	7.922	11.172
600	10	2379.228282	25.609	32.281
600	5	1406.255502	20.422	33.812
800	10	2729.268892	48.281	104.875
800	5	1617.268652	43.688	72.808

Our future research directions include a more detailed study of the application of the branch and bound method to solving k-medoid problems. In particular, we would like to investigate the conditions when the relaxation optimal solution is non-integer. This condition depends on the slope of the objective function and therefore the distances between every pair of points. It is also important to study the interpretation of branching in this problem.

We also would like to study the application of Gomory-Chvátal cuts [6, 10, 13], since this approach looks especially promising. In particular, for fixed n and k the Gomory-Chvátal Closure can be constructed in polynomials time [10]. In most cases, we can stop even before this closure is fully constructed.

Informally, this approach resembles the situation where one needs to solve numerous linear non-homogeneous systems with the same non-singular constraint matrix and different right-hand sides. In general, it is not efficient to solve such systems through matrix inverse, but in this particular settings it may be efficient. Similar situation was studied in [28].

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