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MULTIPLE POSITIVE SOLUTIONS FOR A SYSTEM OF CONFORMABLE FRACTIONAL DIFFERENTIAL EQUATIONS

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Abstract. In this paper, we discuss a new system of conformable fractional differential equations

$$\begin{cases} T_\alpha x(t)+a(t)f(t,y(t))=0, t\in(0,1), \\ T_\beta y(t)+b(t)g(t,x(t))=0, t\in(0,1), \\ x(0)=0, x(1)=\mu\int_0^1 x(t)dt, \mu\in[0,2), \\ y(0)=0, y(1)=\nu\int_0^1 y(t)dt, \nu\in[0,2), \end{cases}$$

where $\alpha,\beta\in(1,2]$, T_α and T_β are the conformable fractional derivatives of orders α and β. By using the fixed point index theory, we obtain the existence of single and multiple positive solutions to the conformable fractional differential system. Moreover, we give two concrete examples to illustrate our main results.

Keywords. Fixed point; Multiple solutions; Conformable fractional derivative; Fixed point index.

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1. Introduction

The fractional differential equations can taken from many fields such as mathematics, engineering and so on. We can also see different fractional differential equations in scientific disciplines which include electron-analytical chemistry, biophysics, signal and image processing, blood flow phenomena, aerodynamics, electrical circuits, nonlinear oscillation of earthquake, etc. We refer the reader to [1–9] and the references cited therein. Several definitions of fractional derivatives, which are Riemann-Liouville, Caputo-type and Hadamard type fractional derivatives, were discussed in the literature.

Recently, a new definition of fractional derivative, called the conformable fractional derivative, was given in [10] due to Khalil et al. introduced in 2014. In 2015, Abdeljawad [11] investigated the conformable fractional calculus and proved chain rules, Gronwall's inequality, Taylor power series expansions and so forth. Recently, many results were investigated on boundary value problems of conformable fractional differential equations, see [12, 13]. For example, by using the notion of tube solution and

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Schauder's fixed-point theorem, Bayour et al. [14] proved the existence of solutions for a fractional nonlinear conformable differential equation with initial condition. Asawasamrit et al in [15] discussed a fractional periodic boundary value problem for impulsive conformable fractional integro-differential equations. They used the method of upper and lower solutions and monotone iterative technique, and then they gave the existence of a unique solution. In [12], by using the method of upper and lower solutions and monotone iterative technique, Meng and Cui obtained the existence of extremal iteration solutions to conformable fractional differential equations involving Riemann-Stieltjes integral boundary conditions. Zhong and Wang [16] discussed a conformable fractional differential equation with an integral boundary condition

$$\begin{cases} T_\alpha u(t)+f(t,u(t))=0, t\in[0,1], \\ u(0)=0, u(1)=\beta\int_0^1 u(t)dt, \beta\in[0,2), \end{cases}$$

where $\alpha\in(1,2]$, T_α denotes the conformable fractional derivative of order α and $f:[0,1]\times[0,+\infty)\to[0,+\infty)$ is a continuous function. The existence of at least one positive solution was given by using a fixed point theorem in a cone.

This paper deals with the existence and multiplicity of positive solutions for integral boundary problems of a new conformable fractional differential equation system

$$\begin{cases} T_\alpha x(t)+a(t)f(t,y(t))=0, t\in(0,1), \\ T_\beta y(t)+b(t)g(t,x(t))=0, t\in(0,1), \\ x(0)=0, x(1)=\mu\int_0^1 x(t)dt, \mu\in[0,2), \\ y(0)=0, y(1)=\nu\int_0^1 y(t)dt, \nu\in[0,2), \end{cases} \tag{1.1}$$

where $\alpha,\beta\in(1,2]$, T_α, T_β stand for the conformable fractional derivatives, and
(H_1) $f,g:[0,1]\times[0,+\infty)\to[0,+\infty)$ are continuous functions;
(H_2) $a(t),b(t)$ are continuous and do not vanish identically on any (c,d) with $0\le c<d\le 1$.

We intend to give several sufficient conditions for the existence of at least one or two positive solutions for system (1.1). The method used here is the fixed point index theory in ordered Banach spaces. As we know, there are still very few papers devoted to the study of positive solutions for conformable fractional differential system with integral boundary conditions. We also find that there is no results of conformable fractional problems obtained by fixed point index theory. So it is worthwhile to investigate system (1.1).

Next, we recall some definitions, notations and known results in Section 2. In Section 3, we provide our main results. The obtained results are well illustrated by two examples in Section 4.

2. Preliminaries

Definition 2.1. [10, 11] Let $\alpha\in(0,1)$. The derivative of a function $f:[0,\infty)\to\mathbb{R}$ defined by

$$T_\alpha f(t)=\lim_{\delta\to 0}\frac{f(t+\delta t^{1-\alpha})-f(t)}{\delta},$$

is called the conformable fractional derivative of order α. If $T_\alpha f(t)$ exists on $(0,b)$, then $T_\alpha f(0)=\lim_{t\to 0}T_\alpha f(t)$.

Definition 2.2. [10, 11] For f: $[0,\infty)\to\mathbb{R}$, the expression

$$I_\alpha f(t)=\frac{1}{n!}\int_0^t (t-s)^n s^{\alpha-n-1}f(s)ds,$$

where $n = [\alpha]$, $[\alpha]$ denotes the integer part of number α, is called the conformable fractional integral of order α.

Lemma 2.3. *[16] Let $h \in C[0,1]$, $\beta \neq 2$ and $1 < \alpha \leq 2$. Then the boundary value problem*

$$\begin{cases} T_\alpha x(t) + h(t) = 0, 0 \leq t \leq 1, \\ x(0) = 0, x(1) = \beta \int_0^1 x(t)dt, \end{cases} \tag{2.1}$$

has a unique solution $x(t) = \int_0^1 G(t,s)h(s)ds$, where

$$G(t,s) = G_1(t,s) + G_2(t,s),$$

$$G_1(t,s) = \begin{cases} (1-t)s^{\alpha-1}, \ 0 \leq s \leq t \leq 1, \\ ts^{\alpha-2}(1-s), \ 0 < t \leq s \leq 1, \end{cases}$$

$$G_2(t,s) = \frac{2\beta t}{2-\beta} \int_0^1 G_1(\tau,s)d\tau.$$

A simple calculation shows that

$$\begin{aligned} G_2(t,s) &= \frac{2\beta t}{2-\beta} \int_0^1 G_1(\tau,s)d\tau \\ &= \frac{2\beta t}{2-\beta} \left[\int_0^s G_1(\tau,s)d\tau + \int_s^1 G_1(\tau,s)d\tau \right] \\ &= \frac{2\beta t}{2-\beta} \left[\int_0^s \tau s^{\alpha-2}(1-s)d\tau + \int_s^1 (1-\tau)s^{\alpha-1}d\tau \right] \\ &= \frac{\beta t s^{\alpha-1}(1-s)(s+1-s)}{2-\beta} \\ &= \frac{\beta t s^{\alpha-1}(1-s)}{2-\beta}. \end{aligned}$$

Lemma 2.4. *[16] Let $G(t,s)$ be given in Lemma 2.3. Then we have the following properties:*
(i) $G(t,s)$ is a continuous function on $(0,1] \times (0,1]$ and

$$0 \leq q(t)G_1(s,s) \leq G_1(t,s) \leq G_1(s,s) \text{ for } (t,s) \in (0,1] \times (0,1], \tag{2.2}$$

$$0 \leq q(t)G_2(1,s) \leq G_2(t,s) \leq G_2(1,s) \text{ for } (t,s) \in (0,1] \times (0,1]; \tag{2.3}$$

(ii) $0 \leq q(t)\theta(s) \leq G(t,s) \leq \theta(s)$ for $(t,s) \in (0,1] \times (0,1]$,
where $q(t) = t(1-t)$ and $\theta(s) = G_1(s,s) + G_2(1,s) = \frac{2s^{\alpha-1}(1-s)}{2-\beta}, \beta \in [0,2)$.

In the sequel, in order to discuss the existence of positive solutions for system (1.1), we present some abstract concepts in ordered Banach spaces and index fixed point theory.

Let $(E, ||\cdot||)$ be a real Banach space and $P \subset E$ be a cone, θ be the zero element of E. Then E has a partially order, i.e., for $u, v \in P$, $v \leq u$ if and only if $u - v \in P$. Define the convex sets P_ρ, $\overline{P_\rho}(\rho > 0)$ by $P_\rho := \{y \in P : ||y|| < \rho\}, \overline{P_\rho} := \{y \in P : ||y|| \leq \rho\}$.

Lemma 2.5. *[17, 18] Let K be a closed convex set in E and let D be a bounded open set such that $D_k := D \bigcap K \neq \emptyset$. Let $T : \overline{D_k} \to K$ be a compact map. Suppose that $x \neq Tx$ for all $x \in \partial D_k$.*
(i) (Existence) If $i(T, D_k, K) \neq 0$, where i denotes the fixed point index. Then T has at least a fixed point in D_k;

(ii) (Additivity) If U_1, U_2 are disjoint relatively open subsets of D_k such that $x \neq Tx$ for $x \in \overline{D_k} \setminus (U_1 \cup U_2)$, then

$$i(T, D_k, K) = i(T, U_1, K) + i(T, U_2, K),$$

where

$$i(T, U_j, K) = i(T|_{\overline{U_j}}, U_j, K)(j = 1, 2).$$

Lemma 2.6. *[17, 18] For $r > 0$, suppose that $T\colon \overline{P_r} \to P$ is a completely continuous operator such that $x \neq Tx$ for $x \in \partial P_r$.*
(i) If $||x|| \leq ||Tx||$ for $x \in \partial P_r$, then $i(T, P_r, P) = 0$;
(ii) If $||x|| \geq ||Tx||$ for $x \in \partial P_r$, then $i(T, P_r, P) = 1$.

Now we utilize Lemmas 2.5 and 2.6 to study system (1.1). Let $E = C[0,1] = \{x : [0,1] \to \mathbb{R}$ is continuous$\}$ with the standard norm $||x|| = \max\{|x(t)| : t \in [0,1]\}$. We consider system (1.1) in $E \times E$ with $||(x,y)|| = ||x|| + ||y||$ for $(x,y) \in E \times E$. It is clear that $(E \times E, ||(\cdot,\cdot)||)$ is a Banach space. Taking $\varepsilon \in (0, \frac{1}{2})$, we define

$$P := \{(x,y) \in E \times E : x(t), y(t) \geq 0, t \in [0,1] \text{ and } \min_{t \in [\varepsilon, 1-\varepsilon]} (x(t) + y(t)) \geq q(t)(||x|| + ||y||), t \in [\varepsilon, 1-\varepsilon]\},$$

where $q(t)$ is defined in Lemma 2.4. Then $P \subset E \times E$ is a cone. From Lemma 2.3, we can get the following.

Lemma 2.7. *Let (H_1) and (H_2) be satisfied. Then $(x,y) \in E \times E$ is a solution of (1.1) if and only if it is a solution of the following system:*

$$\begin{cases} x(t) = \int_0^1 G(t,s)a(s)f(s, y(s))ds, \\ y(t) = \int_0^1 G(t,s)b(s)g(s, x(s))ds, \end{cases}$$

where $G(t,s)$ is given as in Lemma 2.3.

For $(x,y) \in E \times E$, define $T : E \times E \to E \times E$ by

$$T(x,y)(t) = (A_1 x, A_2 y)(t)$$

and $A_1, A_2 : E \to E$ by

$$\begin{cases} A_1 x(t) = \int_0^1 G(t,s)a(s)f(s, y(s))ds, \\ A_2 y(t) = \int_0^1 G(t,s)b(s)g(s, x(s))ds. \end{cases}$$

Evidently, (x,y) is a solution of system (1.1) if and only if it is a fixed point of T.

3. Main results

Set

$$f_0 = \liminf_{(x,y) \to (0^+, 0^+)} \min_{t \in [0,1]} \frac{f(t,y)}{x+y}, g_0 = \liminf_{(x,y) \to (0^+, 0^+)} \min_{t \in [0,1]} \frac{g(t,x)}{x+y},$$
$$f^0 = \limsup_{(x,y) \to (0^+, 0^+)} \max_{t \in [0,1]} \frac{f(t,y)}{x+y}, g^0 = \limsup_{(x,y) \to (0^+, 0^+)} \max_{t \in [0,1]} \frac{g(t,x)}{x+y},$$
$$f_\infty = \liminf_{(x,y) \to (+\infty, +\infty)} \min_{t \in [0,1]} \frac{f(t,y)}{x+y}, g_\infty = \liminf_{(x,y) \to (+\infty, +\infty)} \min_{t \in [0,1]} \frac{g(t,x)}{x+y},$$
$$f^\infty = \limsup_{(x,y) \to (+\infty, +\infty)} \max_{t \in [0,1]} \frac{f(t,y)}{x+y}, g^\infty = \limsup_{(x,y) \to (+\infty, +\infty)} \max_{t \in [0,1]} \frac{g(t,x)}{x+y},$$

$$\lambda_1 = \max\left\{\frac{3}{4}\{q^2(\varepsilon)\int_{\varepsilon}^{1-\varepsilon}\theta(s)a(s)ds\}^{-1}, \frac{3}{4}\{q^2(\varepsilon)\int_{\varepsilon}^{1-\varepsilon}\theta(s)b(s)ds\}^{-1}\right\},$$

$$\lambda_2 = \min\left\{\frac{1}{4}\{\int_0^1\theta(s)a(s)ds\}^{-1}, \frac{1}{4}\{\int_0^1\theta(s)b(s)ds\}^{-1}\right\},$$

where $\theta(s)$ is defined in Lemma 2.4.

Lemma 3.1. *Let $(H_1),(H_2)$ be satisfied. Then $T\colon P\to P$ is a completely continuous operator.*

Proof. For $(x,y)\in P$, by Lemma 2.4, we have

$$A_1x(t) = \int_0^1 G(t,s)a(s)f(s,y(s))ds \le \int_0^1 \theta(s)a(s)f(s,y(s))ds.$$

Hence, $||A_1x|| \le \int_0^1 \theta(s)a(s)f(s,y(s))ds$. In addition,

$$A_1x(t) \;\; = \;\; \int_0^1 G(t,s)a(s)f(s,y(s))ds \ge q(t)\int_0^1 \theta(s)a(s)f(s,y(s))ds \ge q(t)||A_1x||.$$

Using the same way, we can get $A_2y(t)\ge q(t)||A_2y||$ for $(x,y)\in P$. So

$$A_1x(t)+A_2y(t)\ge q(t)(||A_1x||+||A_2y||) \text{ for } (x,y)\in P,$$

and

$$\min_{t\in[\varepsilon,1-\varepsilon]}\{A_1x(t)+A_2y(t)\}\ge q(t)(||A_1x||+||A_2y||), t\in[\varepsilon,1-\varepsilon].$$

These yield that $T:P\to P$. Next we denote the operator

$$A_1 := A_{11}+A_{12}, \tag{3.1}$$

where

$$A_{11}x(t) = \int_0^1 G_1(t,s)a(s)f(s,y(s))ds, \tag{3.2}$$

and

$$A_{12}x(t) = \int_0^1 G_2(t,s)a(s)f(s,y(s))ds. \tag{3.3}$$

Let $K=\{x\in C[0,1], x(t)\ge 0, t\in[0,1]\}$. Using (2.2) and (2.3), we can easily prove that $A_{11}(K)\subset K$ and $A_{12}(K)\subset K$. Further, we observe that $G_1(t,s)$ is singular on $[0,1]\times[0,1]$. So the complete continuity of the operator A_{11} can be verified by using approximations of oparators (see [19]). In addition, we can easily check that $A_{12}\colon K\to C[0,1]$ is completely continuous by using Ascoli-Arzela theorem. So $A_1:K\to K$ is a completely continuous operator. By the same way, we know that A_2 is also completely continuous. Since $P\subseteq K\times K$ and $T(x,y)(t)=(A_1x,A_2y)(t)$, we can prove that $T:P\to P$ is completely continuous. □

Theorem 3.2. *Suppose that $(H_1),(H_2)$ hold and $f_0, g_0>\lambda_1$; $f^\infty, g^\infty<\lambda_2$ with $0<\lambda_1,\lambda_2<\infty$. Then system (1.1) has at least one positive solution.*

Proof. Since f_0 and $g_0>\lambda_1$, there exists a number $r_1>0$ such that

$$f(t,y)\ge\lambda_1(x+y), g(t,x)\ge\lambda_1(x+y) \text{ for } 0\le x,y,x+y\le r_1.$$

Let $P_{r_1} = \{(x,y) \in P : ||(x,y)|| < r_1\}$. For $(x,y) \in \partial P_{r_1}$, we know $||(x,y)|| = ||x|| + ||y|| = r_1$ and $\min\limits_{t\in[\varepsilon,1-\varepsilon]}(x(t)+y(t)) \geq q(t)(||x||+||y||)$, $t \in [\varepsilon, 1-\varepsilon]$. Lemma 2.4 implies

$$\begin{aligned} A_1x(t) &= \int_0^1 G(t,s)a(s)f(s,y(s))ds \geq \int_\varepsilon^{1-\varepsilon} G(t,s)a(s)f(s,y(s))ds \\ &\geq \lambda_1 q(t)\int_\varepsilon^{1-\varepsilon} \theta(s)a(s)(x(s)+y(s))ds \geq \lambda_1 q(t)\int_\varepsilon^{1-\varepsilon} q(s)\theta(s)a(s)ds(||x||+||y||) \\ &\geq \lambda_1 q(t)q(\varepsilon)\int_\varepsilon^{1-\varepsilon} \theta(s)a(s)ds(||x||+||y||). \end{aligned}$$

So,

$$\begin{aligned} ||A_1x|| &= \max_{t\in[0,1]} A_1x(t) \geq \max_{t\in[\varepsilon,1-\varepsilon]} A_1x(t) \geq \max_{t\in[\varepsilon,1-\varepsilon]} \lambda_1 q(t)q(\varepsilon)\int_\varepsilon^{1-\varepsilon}\theta(s)a(s)ds(||x||+||y||) \\ &\geq \lambda_1 q^2(\varepsilon)\int_\varepsilon^{1-\varepsilon}\theta(s)a(s)ds(||x||+||y||) \geq \frac{3}{4}(||x||+||y||) > \frac{1}{2}||(x,y)||. \end{aligned}$$

Similarly, we get $||A_2y|| > \frac{1}{2}||(x,y)||$ for $(x,y) \in \partial P_{r_1}$. Hence, $||T(x,y)|| = ||A_1x|| + ||A_2y|| > ||(x,y)||$, $(x,y) \in \partial P_{r_1}$. From Lemma 2.6 (i), we get

$$i(T, P_{r_1}, P) = 0. \tag{3.4}$$

Next, if f^∞ and $g^\infty < \lambda_2$, there is $R > r_1$ such that

$$f(t,y) \leq \lambda_2(x+y), g(t,x) \leq \lambda_2(x+y) \text{ for } x+y \geq R.$$

Take $r_2 > \frac{R}{q(\varepsilon)}$, where $q(\varepsilon) = \varepsilon(1-\varepsilon) \in (0, \frac{1}{4})$ and let $P_{r_2} = \{(x,y) \in P : ||(x,y)|| < r_2\}$. For $(x,y) \in \partial P_{r_2}$, we have $\min\limits_{t\in[\varepsilon,1-\varepsilon]}(x(t)+y(t)) \geq q(t)(||x||+||y||) \geq q(\varepsilon)r_2 > R, t \in [\varepsilon, 1-\varepsilon]$. By using Lemma 2.4, we have

$$\begin{aligned} A_1x(t) &= \int_0^1 G(t,s)a(s)f(s,y(s))ds \leq \lambda_2\int_0^1 \theta(s)a(s)(x(s)+y(s))ds \\ &\leq \lambda_2\int_0^1 \theta(s)a(s)ds(||x||+||y||) \leq \frac{1}{4}||(x,y)|| < \frac{1}{2}||(x,y)||. \end{aligned}$$

Hence, $||A_1x|| < \frac{1}{2}||(x,y)||$ for $(x,y) \in \partial P_{r_2}$. Similarly, we get $||A_2y|| < \frac{1}{2}||(x,y)||$ for $(u,v) \in \partial P_{r_2}$. Thus $||T(x,y)|| = ||A_1x|| + ||A_2y|| < ||(x,y)||, (x,y) \in \partial P_{r_2}$. From Lemma 2.6 (ii), we obtain

$$i(T, P_{r_2}, P) = 1. \tag{3.5}$$

Noting that $r_1 < r_2$, it follows from the (ii) in Lemma 2.5 and (3.4)-(3.5) that

$$i(T, P_{r_2} \setminus \overline{P_{r_1}}, P) = i(T, P_{r_2}, P) - i(T, P_{r_1}, P) = 1.$$

From Lemma 2.5 (i), we know T has a fixed point (x,y) in $P_{r_2} \setminus \overline{P_{r_1}}$. It is obvious that (x,y) is a positive solution for system (1.1) with $r_1 < ||(x,y)|| < r_2$. □

Corollary 3.3. *Assume that* $(H_1), (H_2)$ *hold and* $f_0, g_0 = \infty; f^\infty, g^\infty = 0$. *Then system (1.1) has at least one positive solution.*

Corollary 3.4. *Assume that* $(H_1), (H_2)$ *hold and* $f_0, g_0 = \infty; f^\infty, g^\infty < \lambda_2$ *with* $0 < \lambda_2 < \infty$. *Then system (1.1) has at least one positive solution.*

Corollary 3.5. *Assume that* $(H_1),(H_2)$ *hold and* $f_0, g_0 > \lambda_1$ *with* $0 < \lambda_1 < \infty$, $f^\infty, g^\infty = 0$. *Then system (1.1) has at least one positive solution.*

Theorem 3.6. *Suppose that* $(H_1),(H_2)$ *hold and* $f_\infty, g_\infty > \lambda_1$; $f^0, g^0 < \lambda_2$ *with* $0 < \lambda_1, \lambda_2 < \infty$. *Then system (1.1) has at least one positive solution.*

Proof. Since f^0 and $g^0 < \lambda_2$, there exists a number $l_1 > 0$ such that

$$f(t,y) \le \lambda_2(x+y), g(t,x) \le \lambda_2(x+y) \text{ for } 0 \le x,y,x+y \le l_1.$$

Let $P_{l_1} = \{(x,y) \in P : ||(x,y)|| < l_1\}$. For $(x,y) \in \partial P_{l_1}$, by using the same calculation in the proof of Theorem 3.2, for $t \in [0,1]$, Lemma 2.4 implies $||A_1 x|| < \frac{1}{2}||(x,y)||$ for $(x,y) \in \partial P_{l_1}$. Similarly, we get $||A_2 y|| < \frac{1}{2}||(x,y)||$ for $(x,y) \in \partial P_{l_1}$. Thus $||T(x,y)|| = ||A_1 x|| + ||A_2 y|| < ||(x,y)||$, $(x,y) \in \partial P_{l_1}$. Lemma 2.6 (ii) implies

$$i(T, P_{l_1}, P) = 1. \tag{3.6}$$

Next, we consider f_∞ and $g_\infty > \lambda_1$. There is $R > l_1$ such that

$$f(t,y) \ge \lambda_1(x+y), g(t,x) \ge \lambda_1(x+y) \text{ for } x+y \ge R.$$

Take $l_2 > \frac{R}{q(\varepsilon)}$, where $q(\varepsilon) = \varepsilon(1-\varepsilon) \in (0, \frac{1}{4})$ and let $P_{l_2} = \{(x,y) \in P : ||(x,y)|| < l_2\}$. Then for $(x,y) \in \partial P_{l_2}$, there is $\min\limits_{t\in[\varepsilon,1-\varepsilon]}(x(t)+y(t)) \ge q(t)(||x||+||y||) \ge q(\varepsilon) l_2 > R$ for $t \in [\varepsilon, 1-\varepsilon]$. We also have $||T(x,y)|| = ||A_1 x|| + ||A_2 y|| > ||(x,y)||$, $(x,y) \in \partial P_{l_2}$. Thus, Lemma 2.6 (i) implies

$$i(T, P_{l_2}, P) = 0. \tag{3.7}$$

Noting that $l_1 < l_2$, it follows from the (ii) in Lemma 2.5 and (3.6)-(3.7) that

$$i(T, P_{l_2} \setminus \overline{P_{l_1}}, P) = i(T, P_{l_2}, P) - i(T, P_{l_1}, P) = -1.$$

Thus, from Lemma 2.5 (i), we know T has a fixed point (x,y) in $P_{l_2} \setminus \overline{P_{l_1}}$. Evidently, (x,y) is a positive solution for system (1.1) with $l_1 < ||(x,y)|| < l_2$. □

Corollary 3.7. *Assume that* $(H_1),(H_2)$ *hold and* $f_\infty, g_\infty = \infty; f^0, g^0 = 0$. *Then system (1.1) has at least one positive solution.*

Corollary 3.8. *Assume that* $(H_1),(H_2)$ *hold and* $f_\infty, g_\infty = \infty; f^0, g^0 < \lambda_2$ *with* $0 < \lambda_2 < \infty$. *Then system (1.1) has at least one positive solution.*

Corollary 3.9. *Assume that* $(H_1),(H_2)$ *hold and* $f_\infty, g_\infty > \lambda_1$ *with* $0 < \lambda_1 < \infty$, $f^0, g^0 = 0$. *Then system (1.1) has at least one positive solution.*

Theorem 3.10. *Suppose that* $(H_1),(H_2)$ *hold and the following conditions are satisfied:*
(H_3) $f_0, g_0 > \lambda_1$ *and* $f_\infty, g_\infty = \infty$;
(H_4) *there exists a constant* $\rho > 0$ *such that*

$$f(t,y) \le \lambda_2 \rho, g(t,x) \le \lambda_2 \rho \text{ for } t \in [0,1] \text{ and } x,y,x+y \in [0,\rho].$$

Then system (1.1) has at least two positive solutions (x_1,y_1) *and* (x_2,y_2) *such that* $0 < ||(x_1,y_1)|| < \rho < ||(x_2,y_2)||$.

Proof. Since f_0 and $g_0 > \lambda_1$, there is a small number $0 < \gamma_1 < \rho$ such that

$$f(t,y) \geq \lambda_1(x+y), g(t,x) \geq \lambda_1(x+y) \text{ for } 0 \leq x,y,x+y \leq \gamma_1.$$

Set $P_{\gamma_1} = \{(x,y) \in P : ||(x,y)|| < \gamma_1\}$. Similar to the proof of Theorem 3.2, we get $||T(x,y)|| = ||A_1x|| + ||A_2y|| > ||(x,y)||$, $(x,y) \in \partial P_{\gamma_1}$. From Lemma 2.6 (i), we get

$$i(T,P_{\gamma_1},P) = 0. \tag{3.8}$$

Next, let $P_\rho = \{(x,y) \in P : ||(x,y)|| < \rho\}$. For $(x,y) \in \partial P_\rho$, Lemma 2.4 and (H_4) imply that

$$\begin{aligned} A_1x(t) &= \int_0^1 G(t,s)a(s)f(s,y(s))ds \leq \int_0^1 \theta(s)a(s)\lambda_2\rho ds \\ &= \lambda_2\rho\int_0^1 \theta(s)a(s)ds \leq \frac{\rho}{4} < \frac{1}{2}||(x,y)||. \end{aligned}$$

Hence, $||A_1x|| < \frac{1}{2}||(x,y)||$ for $(x,y) \in \partial P_\rho$. Similarly, we can prove $||A_2y|| < \frac{1}{2}||(x,y)||$ for $(x,y) \in \partial P_\rho$. Thus $||T(x,y)|| = ||A_1x|| + ||A_2y|| < ||(x,y)||, (x,y) \in \partial P_\rho$. Lemma 2.6 (ii) implies

$$i(T,P_\rho,P) = 1. \tag{3.9}$$

Finally, from f_∞ and $g_\infty = \infty$, we have that there exists $R > \rho$ such that

$$f(t,y) \geq \lambda_1(x+y), g(t,x) \geq \lambda_1(x+y) \text{ for } x+y \geq R.$$

Taking $\gamma_2 > \frac{R}{q(\varepsilon)}$, where $q(\varepsilon) = \varepsilon(1-\varepsilon) \in (0,\frac{1}{4})$, we have $\min\limits_{t\in[\varepsilon,1-\varepsilon]}(x(t)+y(t)) \geq q(t)(||x||+||y||) \geq q(\varepsilon)\gamma_2 > R$ for $t \in [\varepsilon,1-\varepsilon]$. Let $P_{\gamma_2} = \{(x,y) \in P : ||(x,y)|| < \gamma_2\}$. For $(x,y) \in \partial P_{\gamma_2}$, using Lemma 2.4 and the same calculation of getting (3.4), we obtain $||T(x,y)|| = ||A_1x|| + ||A_2y|| > ||(x,y)||, (x,y) \in \partial P_{\gamma_2}$. From Lemma 2.6 (i), we obtain

$$i(T,P_{\gamma_2},P) = 0. \tag{3.10}$$

Noting that $\gamma_1 < \rho < \gamma_2$, it follows from (3.8)-(3.10) and Lemma 2.5 (ii) that

$$i(T,P_\rho\backslash\overline{P_{\gamma_1}},P) = i(T,P_\rho,P) - i(T,P_{\gamma_1},P) = 1,$$

and

$$i(T,P_{\gamma_2}\backslash\overline{P}_\rho,P) = i(T,P_{\gamma_2},P) - i(T,P_\rho,P) = -1.$$

Consequently, from Lemma 2.5 (i), we know that T has two points (x_1,y_1) and (x_2,y_2) with $(x_1,y_1) \in P_\rho\backslash\overline{P_{\gamma_1}}$ and $(x_2,y_2) \in P_{\gamma_2}\backslash\overline{P}_\rho$. That is, they are positive solutions of system (1.1) which satisfy $0 < ||(x_1,y_1)|| < \rho < ||(x_2,y_2)||$. □

Corollary 3.11. *Assume that $(H_1),(H_2)$ hold and the following conditions are satisfied:*
$(H_3^{'})$ $f_0,g_0 = \infty$ *and* $f_\infty,g_\infty = \infty$;
$(H_4^{'})$ *there exists a constant* $\rho > 0$ *such that*

$$f(t,y) \leq \lambda_2\rho, g(t,x) \leq \lambda_2\rho \textit{ for } t \in [0,1] \textit{ and } x,y,x+y \in [0,\rho].$$

Then system (1.1) has at least two positive solutions (x_1,y_1) and (x_2,y_2) such that $0 < ||(x_1,y_1)|| < \rho < ||(x_2,y_2)||$.

Theorem 3.12. *Suppose that* $(H_1),(H_2)$ *hold and the following conditions are satisfied:*
(H_5) $f^0, g^0 < \lambda_2$ *and* $f^\infty, g^\infty = 0$;
(H_6) *there exists a constant* $\rho' > 0$ *such that*

$$f(t,y) \geq q(\varepsilon)\lambda_1\rho', g(t,x) \geq q(\varepsilon)\lambda_1\rho' \text{ for } t \in [\varepsilon, 1-\varepsilon] \text{ and } x+y \in [q(\varepsilon)\rho', \rho'].$$

Then system (1.1) has at least two positive solutions (x_1,y_1) *and* (x_2,y_2) *such that* $0 < ||(x_1,y_1)|| < \rho' < ||(x_2,y_2)||$.

Proof. Since f^0 and $g^0 < \lambda_2$, there exists a small number $\rho_1 \in (0,\rho')$ such that

$$f(t,y) \leq \lambda_2(x+y), g(t,x) \leq \lambda_2(x+y) \text{ for } 0 \leq x,y,x+y \leq \rho_1.$$

Set $P_{\rho_1} = \{(x,y) \in P : ||(x,y)|| < \rho_1\}$. Then for any $(x,y) \in \partial P_{\rho_1}$, by using the same calculation in the proof of Theorem 3.2, we have $||T(x,y)|| = ||A_1x|| + ||A_2y|| < ||(x,y)||$, $(x,y) \in \partial P_{\rho_1}$. By applying (ii) in Lemma 2.6, we obtain

$$i(T, P_{\rho_1}, P) = 1. \tag{3.11}$$

Secondly, in view of f^∞ and $g^\infty = 0$, there exists $R > \rho'$ such that

$$f(t,y) \leq \lambda_2(x+y), g(t,x) \leq \lambda_2(x+y) \text{ for } x+y \geq R.$$

We divide the proof into two cases: f is bounded and f is unbounded.

Case (i). Suppose f is bounded, which implies that there exists $M > 0$ such that $f(t,y) \leq M$ for all $y \in [0,+\infty)$. Choose $\rho_2 > \max\{\frac{M}{\lambda_2}, R\}$. For $(x,y) \in \partial P_{\rho_2}$ with $||(x,y)|| = \rho_2$, from Lemma 2.2, we have

$$A_1x(t) = \int_0^1 G(t,s)a(s)f(s,y(s))ds \leq M\int_0^1 \theta(s)a(s)ds \leq \lambda_2\rho_2\int_0^1 \theta(s)a(s)ds \leq \frac{1}{4}\rho_2 < \frac{1}{2}||(x,y)||.$$

Case (ii): Suppose f is unbounded. Since $f : [0,\infty) \times [0,\infty) \longrightarrow [0,\infty)$ is continuous, we know that there exists $\rho_2 > \max\{\rho', \frac{R}{q(\varepsilon)}\}$ such that $f(t,y) \leq f(t,\rho_2)$ for $0 < x,y \leq \rho_2$. Then for $(x,y) \in P$ with $||(x,y)|| = \rho_2$,

$$\begin{aligned} A_1x(t) &= \int_0^1 G(t,s)a(s)f(s,y(s))ds \leq \int_0^1 \theta(s)a(s)f(s,\rho_2)ds \\ &\leq \lambda_1\rho_2\int_0^1 \theta(s)a(s)ds \leq \frac{1}{4}\rho_2 < \frac{1}{2}||(x,y)||. \end{aligned}$$

Hence, in either case, we may always set

$$P_{\rho_2} = \{(x,y) \in P : ||(x,y)|| < \rho_2\},$$

such that $||A_1x|| < \frac{1}{2}||(x,y)||$ for $(x,y) \in \partial P_{\rho_2}$. By using the same way, we can prove $||A_2y|| < \frac{1}{2}||(x,y)||$ for $(x,y) \in \partial P_{\rho_2}$. Consequently, $||T(x,y)|| = ||A_1x|| + ||A_2y|| < ||(x,y)||$, $(x,y) \in \partial P_{\rho_2}$. Therefore, by applying (ii) in Lemma 2.6, we obtain

$$i(T, P_{\rho_2}, P) = 1. \tag{3.12}$$

Finally, let $P_{\rho'} = \{(x,y) \in P : ||(x,y)|| < \rho'\}$. Then for $(x,y) \in \partial P_{\rho'} \subset P$, $\min_{t\in[\varepsilon,1-\varepsilon]}(x(t)+y(t)) \geq q(t)(||x|| + ||y||) \geq q(\varepsilon)\rho', t \in [\varepsilon, 1-\varepsilon]$. So for $(x,y) \in \partial P_{\rho'}$ and $t \in [\varepsilon, 1-\varepsilon]$, by Lemma 2.4 and (H_6), we have

$$\begin{aligned} A_1x(t) &= \int_0^1 G(t,s)a(s)f(s,y(s))ds \geq \int_\varepsilon^{1-\varepsilon} G(t,s)a(s)f(s,y(s))ds \geq q(t)\int_\varepsilon^{1-\varepsilon} \theta(s)a(s)f(s,y(s))ds \\ &\geq q^2(\varepsilon)\lambda_1\rho'\int_\varepsilon^{1-\varepsilon} \theta(s)a(s)ds \geq \frac{3}{4}\rho' > \frac{1}{2}||(x,y)||. \end{aligned}$$

Hence, $||A_1x|| > \frac{1}{2}||(x,y)||$ for $(x,y) \in \partial P_{\rho'}$. By using the same way, we get $||A_2y|| > \frac{1}{2}||(x,y)||$ for $(x,y) \in \partial P_{\rho'}$. Thus $||T(x,y)|| = ||A_1x|| + ||A_2y|| > ||(x,y)||$, $(x,y) \in \partial P_{\rho'}$. By applying (i) in Lemma 2.6, we obtain

$$i(T, P_{\rho'}, P) = 0. \tag{3.13}$$

Note that $\rho_1 < \rho' < \rho_2$, it follows from (3.11)-(3.13) and Lemma 2.5 (ii) that

$$i(T, P_{\rho'} \backslash \overline{P_{\rho_1}}, P) = i(T, P_{\rho'}, P) - i(T, P_{\rho_1}, P) = -1,$$

and

$$i(T, P_{\rho_2} \backslash \overline{P}_{\rho'}, P) = i(T, P_{\rho_2}, P) - i(T, P_{\rho'}, P) = 1.$$

Consequently, T has two points (x_1, y_1) and (x_2, y_2) with $(x_1, y_1) \in P_{\rho'} \backslash \overline{P_{\rho_1}}$ and $(x_2, y_2) \in P_{\rho_2} \backslash \overline{P}_{\rho'}$. Hence, they are positive solutions of system (1.1) which satisfy $0 < ||(x_1, y_1)|| < \rho' < ||(x_2, y_2)||$. □

Corollary 3.13. *Assume that* $(H_1), (H_2)$ *hold and the following conditions are satisfied:*
$(H_5^{'})$ $f^0, g^0 = 0$ *and* $f^\infty, g^\infty = 0$;
$(H_6^{'})$ *there exists a constant* $\rho' > 0$ *such that*

$$f(t,y) \geq q(\varepsilon)\lambda_1\rho', g(t,x) \geq q(\varepsilon)\lambda_1\rho' \text{ for } t \in [\varepsilon, 1-\varepsilon] \text{ and } x+y \in [q(\varepsilon)\rho', \rho'].$$

Then system (1.1) has at least two positive solutions (x_1, y_1) *and* (x_2, y_2) *such that* $0 < ||(x_1, y_1)|| < \rho' < ||(x_2, y_2)||$.

4. Two examples

In this section, we present two examples to illustrate our main results.

Example 4.1. Consider the fractional differential system with boundary value problem:

$$\begin{cases} T_{\frac{3}{2}}x(t) + (1+2t^2)\sqrt[5]{(t+1)y(t)} = 0, t \in (0,1), \\ T_{\frac{3}{2}}y(t) + (1+3t^3)\sqrt[3]{(t+1)x(t)} = 0, t \in (0,1), \\ x(0) = 0, x(1) = \frac{7}{4}\int_0^1 x(t)dt, \\ y(0) = 0, y(1) = \frac{7}{4}\int_0^1 y(t)dt. \end{cases} \tag{4.1}$$

Obviously, system (4.1) fits the framework of (1.1) with $\alpha = \beta = \frac{3}{2}, \mu = \nu = \frac{7}{4}$. Let $a(t) = 1+2t^2$, $b(t) = 1+3t^3$ and $f(t,y) = \sqrt[5]{(t+1)y}$, $g(t,x) = \sqrt[3]{(t+1)x}$. It is easy to check that (H_1) and (H_2) satisfied. Moreover, let $\varepsilon = \frac{1}{3}$, $\theta(s) = 4s^{\frac{1}{2}}(1-s)$, and $q^2(\frac{1}{3}) = \frac{4}{81}$.

$$\begin{aligned} \lambda_1 &= \max\left\{\frac{243}{16}[\int_{\frac{1}{3}}^{\frac{2}{3}} 4s^{\frac{1}{2}}(1-s)(1+2s^2)ds]^{-1}, \frac{243}{16}[\int_{\frac{1}{3}}^{\frac{2}{3}} 4s^{\frac{1}{2}}(1-s)(1+3s^3)ds]^{-1}\right\} \\ &= \max\left\{\frac{243}{16} \times \left[\frac{4\sqrt{3}(8884\sqrt{2}-4936)}{76545}\right]^{-1}, \frac{243}{16} \times \left[\frac{4\sqrt{3}(4364\sqrt{2}-2456)}{40095}\right]^{-1}\right\} \\ &= \frac{243}{16} \times \left[\frac{4\sqrt{3}(4364\sqrt{2}-2456)}{40095}\right]^{-1} \approx 70.965, \end{aligned}$$

$$\begin{aligned}\lambda_2 &= \min\left\{\frac{1}{4}[\int_0^1 4s^{\frac{1}{2}}(1-s)(1+2s^2)ds]^{-1}, \frac{1}{4}[\int_0^1 4s^{\frac{1}{2}}(1-s)(1+3s^3)ds]^{-1}\right\}\\ &= \min\left\{\frac{1}{4}\times\frac{315}{496}, \frac{1}{4}\times\frac{165}{256}\right\} = \min\left\{\frac{315}{1984}, \frac{165}{1024}\right\} = \frac{315}{1984}.\end{aligned}$$

$$f_0 = \liminf_{(x,y)\to(0^+,0^+)} \min_{t\in[0,1]} \frac{\sqrt[5]{(t+1)y}}{x+y} = +\infty > \lambda_1 = 70.965,$$

$$g_0 = \liminf_{(x,y)\to(0^+,0^+)} \min_{t\in[0,1]} \frac{\sqrt[3]{(t+1)x}}{x+y} = +\infty > \lambda_1 = 70.965;$$

$$f^\infty = \limsup_{(x,y)\to(+\infty,+\infty)} \max_{t\in[0,1]} \frac{\sqrt[5]{(t+1)y}}{x+y} = 0 < \lambda_2 = \frac{315}{1984},$$

$$g^\infty = \limsup_{(x,y)\to(+\infty,+\infty)} \max_{t\in[0,1]} \frac{\sqrt[3]{(t+1)x}}{x+y} = 0 < \lambda_2 = \frac{315}{1984}.$$

Hence, the conditions of Corollary 3.3 are satisfied. So system (4.1) has at least one positive solution.

Example 4.2. Consider the fractional differential system with boundary value problem:

$$\begin{cases} T_{\frac{3}{2}}x(t) + t^{-\frac{1}{2}}f(t,y) = 0, t\in(0,1),\\ T_{\frac{3}{2}}y(t) + \frac{1}{1-t}g(t,x) = 0, t\in(0,1),\\ x(0) = 0, x(1) = \frac{7}{4}\int_0^1 x(t)dt,\\ y(0) = 0, y(1) = \frac{7}{4}\int_0^1 y(t)dt,\end{cases} \tag{4.2}$$

where

$$f(t,y) = \begin{cases} 108ty^2, 0\le y\le\frac{1}{3},\\ 12t, \frac{1}{3}\le y\le 10^5,\\ 1.2\times 10^{11}ty^{-2}, y\ge 10^5,\end{cases} \quad \text{and } g(t,x) = \begin{cases} 108tx^2, 0\le x\le\frac{1}{3},\\ 12t, \frac{1}{3}\le x\le 10^5,\\ 1.2\times 10^{11}tx^{-2}, x\ge 10^5.\end{cases}$$

In this example, let $\varepsilon = \frac{1}{3}$, $\theta(s) = 4s^{\frac{1}{2}}(1-s)$, $a(t) = t^{-\frac{1}{2}}, b(t) = \frac{1}{1-t}$, and $q^2(\frac{1}{3}) = \frac{4}{81}$. Obviously, $(H_1), (H_2)$ hold and

$$\begin{aligned}\lambda_1 &= \max\left\{\frac{243}{16}[\int_{\frac{1}{3}}^{\frac{2}{3}} 4s^{\frac{1}{2}}(1-s)s^{-\frac{1}{2}}ds]^{-1}, \frac{243}{16}[\int_{\frac{1}{3}}^{\frac{2}{3}} 4s^{\frac{1}{2}}(1-s)\frac{1}{1-s}ds]^{-1}\right\}\\ &= \max\left\{\frac{243}{16}\times\frac{3}{2}, \frac{243}{16}\times\frac{9(2\sqrt{6}+\sqrt{3})}{56}\right\} = \max\left\{\frac{729}{32}, \frac{2187(2\sqrt{6}+\sqrt{3})}{896}\right\} = \frac{729}{32},\end{aligned}$$

$$\begin{aligned}\lambda_2 &= \min\left\{\frac{1}{4}[\int_0^1 4s^{\frac{1}{2}}(1-s)s^{-\frac{1}{2}}ds]^{-1}, \frac{1}{4}[\int_0^1 4s^{\frac{1}{2}}(1-s)\frac{1}{1-s}ds]^{-1}\right\}\\ &= \min\left\{\frac{1}{4}\times\frac{1}{2}, \frac{1}{4}\times\frac{3}{8}\right\} = \frac{3}{32}.\end{aligned}$$

Take $\rho_1 = \frac{1}{3}, \rho_2 = 10^5$. Then $\rho_1 < \rho_2$ and

(i) $f^0 = \limsup_{(x,y)\to(0^+,0^+)} \max_{t\in[0,1]} \frac{108ty^2}{(x+y)} = 0 < \frac{3}{32}$ and $f^\infty = \limsup_{(x,y)\to(+\infty,+\infty)} \max_{t\in[0,1]} = \frac{1.2\times 10^{11}t}{y^2(x+y)} = 0$;

(ii) set $\rho' = \frac{2}{3}$ and $f(t,y) = 12t \ge 4 > \frac{27}{8} = q(\varepsilon)\lambda_1\rho'$ for $t\in[\frac{1}{3},\frac{2}{3}]$ and $x+y\in[\frac{4}{27},\frac{2}{3}]$.

By using the same way, we can check that $g(t,x)$ has the same properties. So all the conditions of Corollary 3.12 are satisfied. Therefore, system (4.2) has at least two positive solutions (x_1,y_1) and (x_2,y_2) with $0 < ||(x_1,y_1)|| < \frac{2}{3} < ||(x_2,y_2)||$.

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