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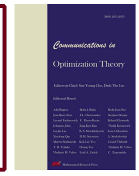

A STOCHASTIC INVENTORY SYSTEM WITH A THRESHOLD BASED PRIORITY SERVICE

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Abstract. In this paper, we consider a (s,Q) inventory system consisting of two parallel queues in which arriving customers belong to any one of the two types such that high priority or low priority served by a single server. The service is given to the customers under threshold based priority scheme in which a threshold L (or control level) is fixed in the first queue. Our aim is to provide sufficient amount of service to low priority customers while providing the best possible service to high priority customers. At the end of each service (i.e., high/low priority), the server decides which queue is to be served next, according to the inventory level and threshold level in the system. For both queues, the arrivals are assumed to be from Poisson processes and the service times have exponentially distributed. The results are illustrated via numerical examples.

Keywords. Inventory system; Perishable goods; Threshold-based control service facility; (s,Q) policy; Markov process.

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1. Introduction

Research on queueing-inventory systems has received much attention over the last decades. An inventory system with positive service time was first studied in [1, 2]. Berman et al. [3] analysed an inventory model with the assumption of both demand, service rates are deterministic, and constant. Consequently, any arrivals during stock out period are placed in the queue. Following [3], Berman and Kim [4] studied a stochastic model for inventory management in which each service requires exactly one item from the inventory. The customer arrival follows Poisson distribution and service time follows exponential distribution. Unlike the deterministic model, they restricted our attention to the instantaneous replenishment i.e., zero lead-time. Berman and Sapna [5] formulated an inventory model with limited waiting space for customers. They assumed that customers arrive according to a Poisson process, service times are arbitrarily distributed and zero lead times. A queueing-inventory system with unlimited waiting space

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for customers was investigated by Berman and Kim [6]. Schwarz et al. [7] have considered $M/M/1$ queueing systems with attached inventory under continuous review. We refer [8–11] for detailed study of Markovian stochastic inventory models with service facility.

The threshold based priority models are well studied and extensively reported in the queueing literature. Lee and Bhaskarsengupta [12] proposed a threshold priority model for ATM networks. They have shown that the model can offer benefits to low priority customers while providing the best service to high priority customers. In other related work, we refer [13–15]. Ishi and Nose [16] investigated the optimal ordering policies for a perishable inventory system with two types of customers under periodic review. According to FIFO issuing policy, stock is depleted by high priority demand first from the newest commodity after ordering and next to low priority demand from the remaining stock.

An important issue in queueing-inventory system with two types of customer is the priority assignment problem. It may be noted that Zhao and Lian [17] analysed a queueing system with an attached inventory in which two classes of customers arrive at a service facility according to Poisson process. Sivakumar and Arivarignan [18] considered a continuous review (s,S) inventory system with two types of customers arrive according to a Markovian arrival process(MAP). Jeganathan et al., [19] generalised a stochastic retrial inventory model under continuous review with non-preemptive priority service. SapnaIsotupa and Samanta [20] studied a continuous review inventory model with lost sales and two types of customers with different shortage costs. They assumed that each class of customer generates unit Poisson demand and the lead time is arbitrarily distributed. Karthik et al., [21] investigated an inventory model with two classes of customers and retrial. They assumed that customers arrive according to Markovian arrival process and the lead time has phase-type distribution. Jeganathan et al., [22] investigated a single server queueing-inventory model with two priority customers and second optional service for high priority customers. Recently, Melikov et al. [23] investigated a queueing inventory system based on the real life situation where the service completion of a each customer who may leave the system with or without purchasing the item. In the later case, they assumed that the customer joins the orbit for decision making. Also, the different sizes of queue and orbit capacity are discussed. Further, we refer to [24–26] for complete analysis of queueing-inventory models with priority customers. We note that in all the above inventory models, if both the types of customer are present in the queue, then the preference was given only to high priority customers than the low priority customers. This induced us to devise a service discipline, which provides adequate service to low priority customers while providing the best quality of service to high priority customers. On threshold based priority scheme, we use the queueing-inventory system to make decisions related to this model. In this paper, we consider a queueing system with inventory management in which two types of customers arriving at a service facility. Each service to type 1 customer uses one item from the inventory whereas the type 2 customers request overhaul of the item. The buffer capacity of each queue is finite and we place a threshold L on the buffer of type 1 customers.

The rest of the paper is systematized as follows. A detailed description of the model is explained in Section 2. The mathematical solution of the model and the steady state analysis of the model is proposed in Section 3. Some key system performance measures are obtained in Section 4. Cost analysis and numerical results are provided in Section 5 to give an insight into the system performance.

2. Model Description

We consider an inventory system with two parallel queues denoted by Q_1 and Q_2 served by a single server, which can stock a maximum of $S(>0)$ units. There are two classes of customers: high priority and low priority customers, which are called type 1 and type 2 customers respectively. The buffer capacity of each queue is limited to accommodate a maximum M number of customers including the one who is getting service. It seems to be reasonable to choose Poisson arrival processes with intensity λ_1 and λ_2 for Q_1 and Q_2 respectively. We have assumed that not all arrivals demand items from the inventory. Each type 1 customer under service needs exactly one item from the inventory where as the type 2 customers request overhaul of the item. The on-hand inventory level decreases by one at the moment of service completion of type 1 customers. The service time follows exponential distribution with parameters μ_1 and μ_2 for type 1 and type 2 customers respectively. The service is under first-come-first-served (FCFS) regime and nonpreemptive. We place a threshold of L on the buffer of type 1 customers. The threshold L has been chosen for this system in order to provide an adequate quality of service to low priority customers while providing the best quality of service to high priority customers. The service policy for this system is a threshold based control, which is rendered according to the following rules.

At each epoch of service completion in Q_1

(1) If the number of customers in Q_1 exceeds the threshold L and the inventory level is positive, a type 1 customer is served.
(2) If the number of customers in Q_1 exceeds L and the inventory level is zero, a type 2 customer is served.
(3) If the number of customers in Q_1 is less than or equal to L and inventory level is positive, the server checks the type of the previous customer that it has served and serves a customer of the other type.
(4) At an instant, if either type of customer is not present when the server is ready for a new service with the positive inventory level, the type of customer that is present is served

The (s,Q) ordering policy is adopted. According to this, when the total of on-hand inventory level downfall to or below the level s, an order for $Q(=S-s>s+1)$ items are placed. The positive lead-time of the replenishment is assumed to be exponential with the rate β. The lifetime of each commodity has a negative exponential distribution with parameter $\gamma>0$. All previously mentioned random variables are independent of each other. In the sequel, I_r refers to an identity matrix of order r, $\mathbf{e}$ refers to a column vector of suitable size having all ones, $[C]_{ij}$ denotes the entry at $(i,j)^{th}$ place of a matrix C, δ is the function defined by $\delta_{jk}=1$ if $j=k$, otherwise $\delta_{jk}=0$, $H(y)$ is the Heaviside function i.e. $H(y)=1$ if $y\geq 0$, otherwise $H(y)=0$, $k\in V_i^j$ means $k=i,i+1,\ldots j$, and

$$\overset{k}{\underset{i=r}{\Omega}} c_i = \begin{cases} c_r c_{r-1}\cdots c_k, & \text{if } r\geq k,\\ 1, & \text{if } r<k. \end{cases}$$

3. Mathematical Solutions of the Model

Let $a(t)$, $c(t)$ and $d(t)$ respectively, denote the inventory level, the number of customers in the queue 1 and the number of customers in the queue 2 at time t, $a(t)\in\{0,1,2,\ldots,S\}$, $c(t)\in\{0,1,2,\ldots,\ldots,M\}$

and $d(t) \in \{0,1,2,\ldots,\ldots,M\}$. Let $b(t)$ denote the server status at time t.

$$b(t) \quad = \quad \begin{cases} 0, & \text{if the server is idle at time } t, \\ 1, & \text{if the server is busy with Type 1 service at time } t, \\ 2, & \text{if the server is busy with Type 2 service at time } t, \end{cases}$$

Thus, the behaviour of this model can be written by a 4-dimensional stochastic process

$$\{f(t) = (a(t), b(t), c(t), d(t)),\ t \geq 0\}$$

with the finite discrete state space H, where

$$\begin{aligned} H \quad = \quad & \{a(t)=0,\ b(t)=0,\ c(t)=0,1,\ldots,M,\ d(t)=0\} \cup \\ & \{a(t)=0,\ b(t)=2,\ c(t)=0,1,\ldots,M,\ d(t)=1,2,\ldots,M,\} \cup \\ & \{a(t)=1,2,\ldots,S,\ b(t)=0,\ c(t)=0,\ d(t)=0\} \cup \\ & \{a(t)=1,2,\ldots,S,\ b(t)=2,\ c(t)=0,\ d(t)=1,2,\ldots,M\} \cup \\ & \{a(t)=1,2,\ldots,S,\ b(t)=1,\ c(t)=1,2,\ldots,M,\ d(t)=0\} \cup \\ & \{a(t)=1,2,\ldots,S,\ b(t)=1,2,\ c(t)=1,2,\ldots,M,\ d(t)=1,2,\ldots,M\} \end{aligned}$$

Due to the conjectures of Poisson arrival processes, exponential distributed service times and replenishment process(exponential), $\{f(t), t \geq 0\}$ is a continuous time Markov chain(CTMC). The infinitesimal generator matrix of $\{f(t), t \geq 0\}$ can be expressed as follows:

$$\Lambda = \begin{array}{c} (\mathbf{S}) \\ (\mathbf{S-1}) \\ \vdots \\ (\mathbf{s+1}) \\ (\mathbf{s}) \\ (\mathbf{s-1}) \\ \vdots \\ (\mathbf{1}) \\ (\mathbf{0}) \end{array} \begin{pmatrix} Y_S & X_S & & & & & & & & \\ & Y_{S-1} & X_{S-1} & & & & & & & \\ & & \cdots & & & & & & & \\ & & & \cdots & Y_{s+1} & X_{s+1} & & & & \\ D & & & & & Y_s & X_s & & & \\ & D & & & & & Y_{s-1} & & & \\ & & \cdots & & & & \cdots & & & \\ & & D & & & & & \cdots & Y_1 & X_1 \\ & & & D_1 & & & & & & Y_0 \end{pmatrix}$$

where $D_1 = [d_{1(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)}]$ is a rectangular matrix of size $(M+1)^2 \times (1+2(M^2+M))$ and its entries are given by

$$d_{1(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)} = \begin{cases} \beta, & \begin{array}{llll} j_1 = Q, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4 \\ i_1 = 0, & i_2 = 0, & i_3 = 0, & i_4 = 0, \\ \\ j_1 = Q, & j_2 = 1, & j_3 = i_3, & j_4 = i_4, \\ i_1 = 0, & i_2 = 0, & i_3 \in V_1^M, & i_4 = 0, \\ \\ j_1 = Q, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 = 0, & i_2 = 2, & i_3 = 0, & i_4 \in V_1^M, \\ \\ j_1 = Q, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 = 0, & i_2 = 2, & i_3 \in V_1^M, & i_4 \in V_1^M, \end{array} \\ \\ 0, & \text{otherwise.} \end{cases}$$

The matrix $D = [d_{(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)}]$ is a square matrix of size $(1+2(M^2+M))$ and it governs the transitions from the states of $(\mathbf{i_1}), i_1 \in V_1^s$, into the states of $(\mathbf{i_1}+\mathbf{Q})$ and their elements are given by

$$d_{(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)} = \begin{cases} \beta, & \begin{array}{llll} j_1 = i_1+Q, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_1^s, & i_2 = 0, & i_3 = 0, & i_4 = 0, \\ \\ j_1 = i_1+Q, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_1^s, & i_2 = 2, & i_3 = 0, & i_4 \in V_1^M, \\ \\ j_1 = i_1+Q, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_1^s, & i_2 = 1, & i_3 \in V_1^M, & i_4 = 0, \\ \\ j_1 = i_1+Q, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_1^s, & i_2 = 1,2, & i_3 \in V_1^M, & i_4 \in V_1^M, \end{array} \\ \\ 0, & \text{otherwise.} \end{cases}$$

The matrix $X_1 = [x_{1\,(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)}]$ is a rectangular matrix of size $(1+2(M^2+M)) \times (M+1)^2$ and it governs the transitions from the states of $(\mathbf{1})$ into the states of $(\mathbf{0})$ and their elements are given by

$$x_{1\,(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)} = \begin{cases} \gamma, & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 = 1, & i_2 = 0, & i_3 = 0, & i_4 = 0, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 = 1, & i_2 = 2, & i_3 = 0, & i_4 \in V_1^M, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = 0, & j_3 = i_3, & j_4 = i_4, \\ i_1 = 1, & i_2 = 1, & i_3 \in V_1^M, & i_4 = 0, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = 2, & j_3 = i_3, & j_4 = i_4, \\ i_1 = 1, & i_2 = 1,2, & i_3 \in V_1^M, & i_4 \in V_1^M, \end{array} \\ \mu_1 & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = 0, & j_3 = i_3 - 1, & j_4 = i_4, \\ i_1 = 1, & i_2 = 1, & i_3 \in V_1^M, & i_4 = 0, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = 2, & j_3 = i_3 - 1, & j_4 = i_4, \\ i_1 = 1, & i_2 = 1, & i_3 \in V_1^M, & i_4 \in V_1^M, \end{array} \\ 0, & \text{otherwise.} \end{cases}$$

The matrices $X_{i_1} = [x_{i_1\,(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)}], i_1 \in V_2^S$ govern the transitions from the states of $(\mathbf{i_1})$ into the states of $(\mathbf{i_1 - 1})$ and hence are square matrices of size $(1+2(M^2+M))$ with the following entries

$$x_{i_1\,(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)} = \begin{cases} i_1\gamma, & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_2^S, & i_2 = 0, & i_3 = 0, & i_4 = 0, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_2^S, & i_2 = 2, & i_3 = 0, & i_4 \in V_1^M, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_2^S, & i_2 = 1, & i_3 \in V_1^M, & i_4 = 0, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_2^S, & i_2 = 1,2, & i_3 \in V_1^M, & i_4 \in V_1^M, \end{array} \\ \mu_1 & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = 0, & j_3 = 0, & j_4 = i_4, \\ i_1 \in V_2^S, & i_2 = 1, & i_3 = 1, & i_4 = 0, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = i_2, & j_3 = i_3 - 1, & j_4 = i_4, \\ i_1 \in V_2^S, & i_2 = 1, & i_3 \in V_2^M, & i_4 = 0, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = 2, & j_3 = i_3 - 1, & j_4 = i_4, \\ i_1 \in V_2^S, & i_2 = 1, & i_3 = 1, & i_4 \in V_1^M, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = 2, & j_3 = i_3 - 1, & j_4 = i_4, \\ i_1 \in V_2^S, & i_2 = 1, & i_3 \in V_2^{L+1}, & i_4 \in V_1^M, \end{array} \\ & \begin{array}{llll} j_1 = i_1 - 1, & j_2 = i_2, & j_3 = i_3 - 1, & j_4 = i_4, \\ i_1 \in V_2^S, & i_2 = 1, & i_3 \in V_{L+2}^M, & i_4 \in V_1^M, \end{array} \\ 0, & \text{otherwise.} \end{cases}$$

The matrix $Y_0 = [y_{0(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)}]$ is a square matrix of size $(M+1)^2$ and it governs the transitions from the states of $(\mathbf{0})$ into the states of $(\mathbf{0})$ and their elements are given by

$$y_{0(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)} = \begin{cases} \lambda_1, & \begin{array}{llll} j_1=i_1, & j_2=i_2, & j_3=i_3+1, & j_4=i_4, \\ i_1=0, & i_2=0, & i_3\in V_0^{M-1}, & i_4=0, \end{array} \\ & \begin{array}{llll} j_1=i_1, & j_2=i_2, & j_3=i_3+1, & j_4=i_4, \\ i_1=0, & i_2=2, & i_3\in V_0^{M-1}, & i_4\in V_1^{M}, \end{array} \\ \lambda_2, & \begin{array}{llll} j_1=i_1, & j_2=2, & j_3=i_3, & j_4=i_4+1, \\ i_1=0, & i_2=0, & i_3\in V_0^{M}, & i_4=0, \end{array} \\ & \begin{array}{llll} j_1=i_1, & j_2=2, & j_3=i_3, & j_4=i_4+1, \\ i_1=0, & i_2=2, & i_3\in V_0^{M}, & i_4\in V_1^{M-1}, \end{array} \\ \mu_2, & \begin{array}{llll} j_1=i_1, & j_2=0, & j_3=i_3, & j_4=i_4-1, \\ i_1=0, & i_2=2, & i_3\in V_0^{M}, & i_4=1, \end{array} \\ & \begin{array}{llll} j_1=i_1, & j_2=i_2, & j_3=i_3, & j_4=i_4-1, \\ i_1=0, & i_2=2, & i_3\in V_0^{M}, & i_4\in V_2^{M}, \end{array} \\ -(\lambda_2+\beta+\lambda_1\bar{\delta}_{i_3M}) & \begin{array}{llll} j_1=i_1, & j_2=i_2, & j_3=i_3, & j_4=i_4, \\ i_1=0, & i_2=0, & i_3\in V_0^{M}, & i_4=0, \end{array} \\ -(\lambda_2\bar{\delta}_{i_4M}+\beta \\ +\lambda_1\bar{\delta}_{i_3M}+\mu_2) & \begin{array}{llll} j_1=i_1, & j_2=i_2, & j_3=i_3, & j_4=i_4, \\ i_1=0, & i_2=2, & i_3\in V_0^{M}, & i_4\in V_1^{M}, \end{array} \\ 0, & \text{otherwise.} \end{cases}$$

Finally, the matrices $Y_{i_1} = [y_{i_1\,(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)}], i_1 \in V_1^S$ represents all transitions within $(\mathbf{i_1})$ and $Y_{i_1}, i_1 \in V_1^S$ are square matrices of order $(1+2(M^2+M))$ with the following elements,

$$y_{i_1\,(i_1,i_2,i_3,i_4),(j_1,j_2,j_3,j_4)} = \begin{cases} \lambda_1, & \begin{array}{llll} j_1=i_1, & j_2=1, & j_3=i_3+1, & j_4=i_4, \\ i_1\in V_1^S, & i_2=0, & i_3=0, & i_4=0, \end{array} \\ & \begin{array}{llll} j_1=i_1, & j_2=i_2, & j_3=i_3+1, & j_4=i_4, \\ i_1\in V_1^S, & i_2=2, & i_3=0, & i_4\in V_1^{M}, \end{array} \end{cases}$$

$$
\begin{cases}
 & \begin{array}{llll} j_1 = i_1, & j_2 = i_2, & j_3 = i_3 + 1, & j_4 = i_4, \\ i_1 \in V_1^S, & i_2 = 1, & i_3 \in V_1^{M-1}, & i_4 = 0, \end{array} \\[2ex]
 & \begin{array}{llll} j_1 = i_1, & j_2 = i_2, & j_3 = i_3 + 1, & j_4 = i_4, \\ i_1 \in V_1^S, & i_2 = 1,2, & i_3 \in V_1^{M-1}, & i_4 \in V_1^M, \end{array} \\[2ex]
\lambda_2, & \begin{array}{llll} j_1 = i_1, & j_2 = 2, & j_3 = i_3, & j_4 = i_4 + 1, \\ i_1 \in V_1^S, & i_2 = 0, & i_3 = 0, & i_4 = 0, \\ j_1 = i_1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4 + 1, \\ i_1 \in V_1^S, & i_2 = 2, & i_3 = 0, & i_4 \in V_1^{M-1}, \end{array} \\[2ex]
 & \begin{array}{llll} j_1 = i_1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4 + 1, \\ i_1 \in V_1^S, & i_2 = 1, & i_3 \in V_1^M, & i_4 = 0, \end{array} \\[2ex]
 & \begin{array}{llll} j_1 = i_1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4 + 1, \\ i_1 \in V_1^S, & i_2 = 1,2, & i_3 \in V_1^M, & i_4 \in V_1^{M-1}, \end{array} \\[2ex]
\mu_2, & \begin{array}{llll} j_1 = i_1, & j_2 = 0, & j_3 = i_3, & j_4 = i_4 - 1, \\ i_1 \in V_1^S, & i_2 = 2, & i_3 = 0, & i_4 = 1, \end{array} \\[2ex]
 & \begin{array}{llll} j_1 = i_1, & j_2 = 0, & j_3 = i_3, & j_4 = i_4 - 1, \\ i_1 \in V_1^S, & i_2 = 2, & i_3 = 0, & i_4 \in V_2^M, \end{array} \\[2ex]
 & \begin{array}{llll} j_1 = i_1, & j_2 = 1, & j_3 = i_3, & j_4 = i_4 - 1, \\ i_1 \in V_1^S, & i_2 = 2, & i_3 \in V_1^M, & i_4 \in V_1^M, \end{array} \\[2ex]
\begin{array}{l} -(\lambda_2 + \lambda_1 + i_1\gamma \\ +H(s-i_1)\beta) \end{array} & \begin{array}{llll} j_1 = i_1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_1^S, & i_2 = 0, & i_3 = 0, & i_4 = 0, \end{array} \\[2ex]
\begin{array}{l} -(\bar{\delta}_{i_4M}\lambda_2 + \bar{\delta}_{i_3M}\lambda_1 + i_1\gamma \\ +\mu_2 + H(s-i_1)\beta) \end{array} & \begin{array}{llll} j_1 = i_1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_1^S, & i_2 = 2, & i_3 \in V_0^M, & i_4 \in V_1^M, \end{array} \\[2ex]
\begin{array}{l} -(\bar{\delta}_{i_4M}\lambda_2 + \bar{\delta}_{i_3M}\lambda_1 + i_1\gamma \\ +\mu_1 + H(s-i_1)\beta) \end{array} & \begin{array}{llll} j_1 = i_1, & j_2 = i_2, & j_3 = i_3, & j_4 = i_4, \\ i_1 \in V_1^S, & i_2 = 1, & i_3 \in V_1^M, & i_4 \in V_0^M, \end{array} \\[2ex]
0, & \text{otherwise.}
\end{cases}
$$

It can be seen from the structure of Λ that the homogeneous Markov process $\{f(t), t \geq 0\}$ on the finite space H is irreducible, aperiodic and persistent non-null. Hence the limiting distribution

$$\psi^{(i_1,i_2,i_3,i_4)} = \lim_{t\to\infty} Pr[a(t)=i_1, b(t)=i_2, c(t)=i_3, d(t)=i_4 | a(0), b(0), c(0), d(0)]$$

exists. So, the steady state distribution vector $\Psi = (\Psi^{(0)}, \Psi^{(1)}, \ldots, \Psi^{(S)})$ is the one and only solution of the linear system

$$\Psi\Lambda = \mathbf{0} \tag{3.1}$$

and the normalization condition

$$\Psi\mathbf{e} = \sum_{(i_1,i_2,i_3,i_4)}\sum\sum\sum \psi^{(i_1,i_2,i_3,i_4)} = 1. \tag{3.2}$$

From equation (1), we obtain the following system of equations:

$$\Psi^{(i_1)}Y_{i_1} + \Psi^{(i_1+1)}X_{i_1+1} = 0, \qquad i_1 = 0,1,2,\ldots,Q-1, \tag{3.3}$$

$$\Psi^{(i_1-Q)}D_1 + \Psi^{(i_1)}Y_{i_1} + \Psi^{(i_1+1)}Y_{i_1+1} = 0, \qquad i_1 = Q, \tag{3.4}$$

$$\Psi^{(i_1-Q)}D + \Psi^{(i_1)}Y_{i_1} + \Psi^{(i_1+1)}X_{i_1+1} = 0, \qquad i_1 = Q+1, Q+2, \ldots, S-1, \tag{3.5}$$

$$\Psi^{(i_1)}Y_{i_1} + \Psi^{(i_1-Q)}D = 0, \qquad i_1 = S, \tag{3.6}$$

Now, we develop a recursive algorithm for the solution of the steady-state equations (1) and (2). The limiting probability vector $\Psi^{(i_1)}$, $i_1 \in V_0^S$ can be determined from the following algorithm.

Step 1. : To find the value of Ψ^Q, first we explain the subsequent system of equations:

$$\Psi^Q\left[\left\{(-1)^Q\sum_{j=0}^{s-1}\left[\left(\mathop{\Omega}_{k=Q}^{s+1-j} X_k Y_{k-1}^{-1}\right) D Y_{S-j}^{-1}\left(\mathop{\Omega}_{l=S-j}^{Q+2} X_l D_{l-1}^{-1}\right)\right]\right\} X_{Q+1} + Y_Q + \left\{(-1)^Q \mathop{\Omega}_{j=Q}^{1} X_j Y_{j-1}^{-1}\right\} D\right] = 0,$$

and

$$\Psi^Q\left[\sum_{i_1=0}^{Q-1}\left((-1)^{Q-i_1}\mathop{\Omega}_{j=Q}^{i_1+1} X_j Y_{j-1}^{-1}\right) + I + \sum_{i_1=Q+1}^{S}\left((-1)^{2Q-i_1+1}\sum_{j=0}^{S-i_1}\left[\left(\mathop{\Omega}_{k=Q}^{s+1-j} X_k Y_{k-1}^{-1}\right) D Y_{S-j}^{-1}\left(\mathop{\Omega}_{l=S-j}^{i_1+1} X_l Y_{l-1}^{-1}\right)\right]\right)\right] e = 1.$$

Step 2. : Next, we calculate the following values of

$$\begin{aligned}
\Omega_{i_1} &= (-1)^{Q-i_1}\Psi^Q \mathop{\Omega}_{j=Q}^{i_1+1} X_j Y_{j-1}^{-1}, && i_1 = Q-1, Q-2, \ldots, 0 \\
&= (-1)^{2Q-i_1+1}\Psi^Q \sum_{j=0}^{S-i_1}\left[\left(\mathop{\Omega}_{k=Q}^{s+1-j} X_k Y_{k-1}^{-1}\right) D Y_{S-j}^{-1}\left(\mathop{\Omega}_{l=S-j}^{i_1+1} X_l Y_{l-1}^{-1}\right)\right], && \\
& && i_1 = S, S-1, \ldots, Q+1 \\
&= I, && i_1 = Q.
\end{aligned}$$

Step 3. : By $\Psi^{(Q)}$ and $\Omega_{i_1}, i_1 = 0,1,\ldots,S$, get the value of $\Psi^{(i_1)}, i_1 = 0,1,\ldots,S$. Explicitly,

$$\Psi^{(i_1)} = \Psi^{(Q)}\Omega_{i_1}, \quad i_1 = 0,1,\ldots,S.$$

4. System Performance Measures

In this section, we develop some measures of system performance in the steady state. Using this, we compute the total expected cost rate.

(i) Expected Inventory Level Δ_I :

$$\begin{aligned}\Delta_I &= \sum_{i_1=1}^{S} i_1\psi^{(i_1,0,0,0)} + \sum_{i_1=1}^{S}\sum_{i_4=1}^{M} i_1\psi^{(i_1,2,0,i_4)}\\ &\quad + \sum_{i_1=1}^{S}\sum_{i_3=1}^{M} i_1\psi^{(i_1,1,i_3,0)} + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{M}\sum_{i_4=1}^{M} i_1\psi^{(i_1,i_2,i_3,i_4)}.\end{aligned}$$

(ii) Mean Reorder Rate Δ_R:

$$\begin{aligned}\Delta_R &= (s+1)\gamma\psi^{(s+1,0,0,0)} + \sum_{i_4=1}^{M}(s+1)\gamma\psi^{(s+1,2,0,i_4)} + \sum_{i_3=1}^{M}\{(s+1)\gamma+\mu_1\}\psi^{(s+1,1,i_3,0)}\\ &\quad + \sum_{i_3=1}^{M}\sum_{i_4=1}^{M}\{(s+1)\gamma+\mu_1\}\psi^{(s+1,1,i_3,i_4)} + \sum_{i_3=1}^{M}\sum_{i_4=1}^{M}(s+1)\gamma\psi^{(s+1,2,i_3,i_4)}.\end{aligned}$$

(iii) Mean Perishable Rate Δ_P:

$$\begin{aligned}\Delta_P &= \sum_{i_1=1}^{S} i_1\gamma\psi^{(i_1,0,0,0)} + \sum_{i_1=1}^{S}\sum_{i_4=1}^{M} i_1\gamma\psi^{(i_1,2,0,i_4)}\\ &\quad + \sum_{i_1=1}^{S}\sum_{i_3=1}^{M} i_1\gamma\psi^{(i_1,1,i_3,0)} + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{M}\sum_{i_4=1}^{M} i_1\gamma\psi^{(i_1,i_2,i_3,i_4)}.\end{aligned}$$

(iv) Average Number of Type 1 Customers Lost Δ_{L_1}:

$$\begin{aligned}\Delta_{L_1} &= \lambda_1\psi^{(0,0,M,0)} + \sum_{i_4=1}^{M}\lambda_1\psi^{(0,2,M,i_4)} + \sum_{i_1=1}^{S}\lambda_1\psi^{(i_1,1,M,0)}\\ &\quad + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_4=1}^{M}\lambda_1\psi^{(i_1,i_2,M,i_4)}.\end{aligned}$$

(v) Average Number of Type 2 Customers Lost Δ_{L_2}:

$$\Delta_{L_2} = \sum_{i_3=0}^{M}\lambda_2\psi^{(0,2,i_3,M)} + \sum_{i_1=1}^{S}\lambda_2\psi^{(i_1,2,0,M)} + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{M}\lambda_2\psi^{(i_1,i_2,i_3,i_4)}.$$

(vi) Average Number of Customers in the Queue 1 Δ_{Q_1}:

$$\begin{aligned}\Delta_{Q_1} &= \sum_{i_3=1}^{M} i_3\psi^{(0,0,i_3,0)} + \sum_{i_3=1}^{M}\sum_{i_4=1}^{M} i_3\psi^{(0,2,i_3,i_4)}\\ &\quad + \sum_{i_1=1}^{S}\sum_{i_3=1}^{M} i_3\psi^{(i_1,1,i_3,0)} + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{M}\sum_{i_4=1}^{M} i_3\psi^{(i_1,i_2,i_3,i_4)}.\end{aligned}$$

(vii) Average Number of Customers in the Queue 2 Δ_{Q_2}:

$$\begin{aligned}\Delta_{Q_2} &= \sum_{i_3=0}^{M}\sum_{i_4=1}^{M} i_4\psi^{(0,2,i_3,i_4)} + \sum_{i_1=1}^{S}\sum_{i_4=1}^{M} i_4\psi^{(i_1,2,0,i_4)}\\ &\quad + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{M}\sum_{i_4=1}^{M} i_4\psi^{(i_1,i_2,i_3,i_4)}.\end{aligned}$$

(viii) Average Number of Type 1 Customers Entering into the Queue 1 Δ_{A_1} :

$$\Delta_{A_1} = \sum_{i_3=0}^{M-1} \lambda_1 \psi^{(0,0,i_3,0)} + \sum_{i_3=0}^{M-1}\sum_{i_4=1}^{M} \lambda_1 \psi^{(0,2,i_3,i_4)} + \sum_{i_1=1}^{S} \lambda_1 \psi^{(i_1,0,0,0)} + \sum_{i_1=1}^{S}\sum_{i_4=1}^{M} \lambda_1 \psi^{(i_1,2,0,i_4)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{M-1} \lambda_1 \psi^{(i_1,1,i_3,0)} + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{M-1}\sum_{i_4=1}^{M} \lambda_1 \psi^{(i_1,i_2,i_3,i_4)}.$$

(ix) Average Number of Type 2 Customers Entering into the Queue 2 Δ_{A_2} :

$$\Delta_{A_2} = \sum_{i_3=1}^{M} \lambda_2 \psi^{(0,0,i_3,0)} + \sum_{i_3=0}^{M}\sum_{i_4=1}^{M-1} \lambda_2 \psi^{(0,2,i_3,i_4)} + \sum_{i_1=1}^{S} \lambda_2 \psi^{(i_1,0,0,0)} + \sum_{i_1=1}^{S}\sum_{i_4=1}^{M-1} \lambda_2 \psi^{(i_1,2,0,i_4)} + \sum_{i_1=1}^{S}\sum_{i_3=1}^{M} \lambda_2 \psi^{(i_1,1,i_3,0)} + \sum_{i_1=1}^{S}\sum_{i_2=1}^{2}\sum_{i_3=1}^{M}\sum_{i_4=1}^{M-1} \lambda_2 \psi^{(i_1,i_2,i_3,i_4)}.$$

(x) Expected number of customers entering station 2(η_{W1}):

$$\eta_{W1} = \frac{\Delta_{A_1}}{\eta_{A_1}}.$$

(xi) Expected number of customers entering station 2(η_{W2}):

$$\eta_{W2} = \frac{\Delta_{A_2}}{\eta_{A_2}}.$$

5. Cost Analysis and Sensitivity Investigation

Here we define different costs as

c_h = The inventory carrying cost per unit item per unit time.
c_s = Setup cost per order.
c_p = Failure cost per unit item per unit time.
c_{w_1} = Waiting time cost of a type 1 customer per unit time.
c_{w_2} = Waiting time cost of a type 2 customer per unit time.
c_{l_1} = Cost due to loss of a type 1 customer per unit time.
c_{l_2} = Cost due to loss of a type 2 customer per unit time.

We introduce a cost function, defined as the expected total cost (TC) of the system, given by

$$TC(S,s,L,M) = c_h\Delta_I + c_s\Delta_R + c_p\Delta_P + c_{w_1}\eta_{W1} + c_{w_2}\eta_{W2} + c_{l_1}\Delta_{L_1} + c_{l_2}\Delta_{L_2}. \tag{5.1}$$

5.1. **Sensitivity Investigation.** In this section, we present four interesting numerical examples that qualitatively describe the performance measures of the model under study. In all the examples, we use the cost values of $c_h = 1$, $c_s = 50$, $c_p = 1$, $c_{l_1} = 2$, $c_{l_2} = 3$, $c_{w_1} = 3$, $c_{w_2} = 6$ and fixed parameter values are $\lambda_1 = 2$, $\lambda_2 = 1$, $\beta = 0.1$, $\gamma = 0.001$, $\mu_1 = 4$, $\mu_2 = 3$, $S = 42$, $s = 5$, $L = 3$, $M = 10$, considered.

Example 5.1.

From Figs. 1 to 6, we observe the impact of various parameters on Expected waiting time of high priority customers (EW1) given below:

(1) Expected waiting time of high priority customer (EW1) decreases as β and μ_1 increases but γ and λ_1 decreases.

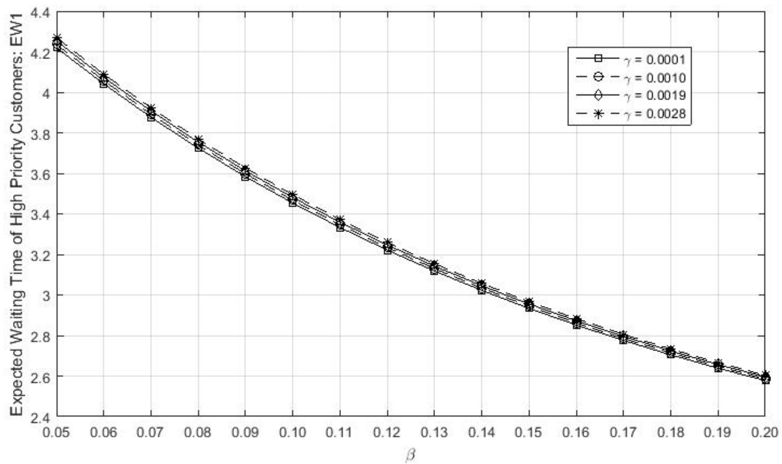

FIGURE 1. $EW1$ vs β for different values of γ

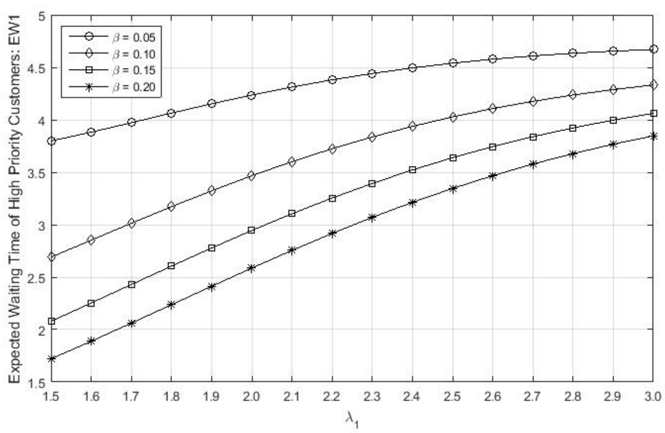

FIGURE 2. $EW1$ vs λ_1 for different values of β

(2) With reference of given ranges of β and λ_1, the rate of expected waiting time of high priority customer is minimum at the higher value of β and λ_1.
(3) With reference of given ranges of all parameters, the rate of expected waiting time of high priority customer is minimum at the higher value of β, λ_1 and μ_1 but the lower value of γ.
(4) Expected waiting time of high priority customer is more sensitive when λ_1 decreases and μ_1 decreases but $\lambda_1 < \mu_1$. Because the expected number of high priority customer beyond the threshold level comes often when λ_1 increases to certain specific level and also, the service time of high priority customer is less than the service time of low priority customer.
(5) On the whole discussion β is highly significant parameter. But the comparison of other parameters γ is more effective.

Example 5.2.

From Figs.7 to 12, we observe the impact of various parameters on Expected waiting time of low priority customer (EW2) given below:

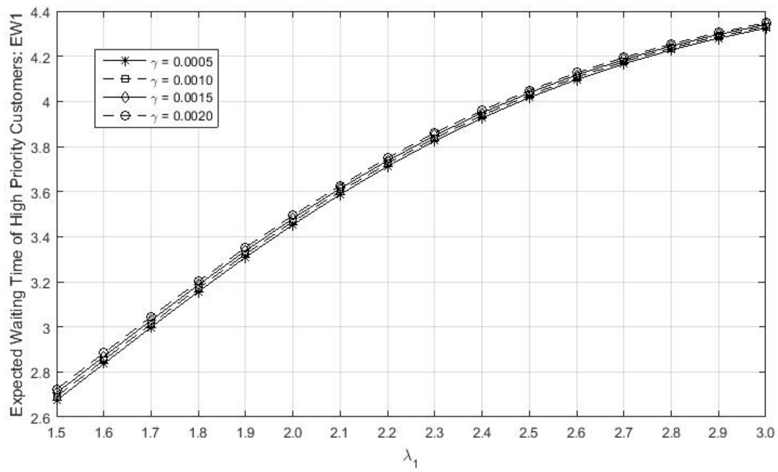

FIGURE 3. $EW1$ vs λ_1 for different values of γ

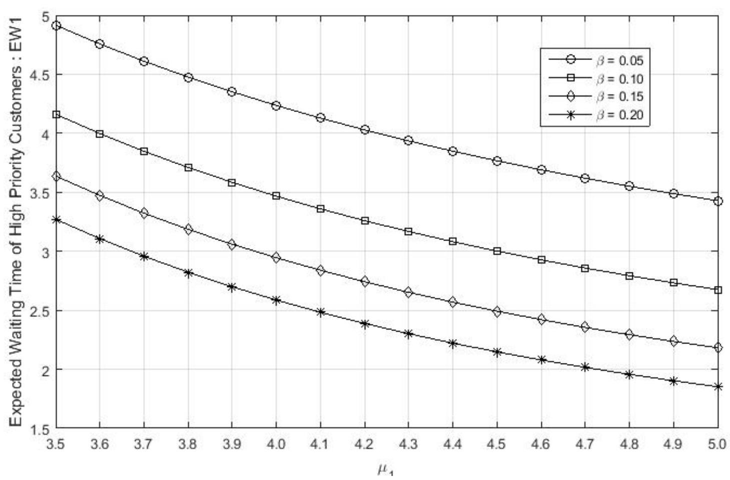

FIGURE 4. $EW1$ vs μ_1 for different values of β

(1) Since the service time of high priority customer is less than the service time of low priority customer and the arrival rate of high priority customer is more than the arrival rate of low priority customer but $\lambda_1 < \mu_1$ and $\lambda_2 < \mu_2$, the service rate of low priority customer is greater than the arrival rate of high priority customer and the service rate of high priority customer is greater than arrival rate of low priority customer.
(2) The expected waiting time of low priority customer (EW2) decreases as μ_2 increases but λ_2 decreases.
(3) The expected number of high priority customer beyond the threshold level does not come when β increases to some specific level and/or γ decreases to some other level. Hence, expected waiting time of low priority customer (EW2) decreases as β increases and γ decreases.
(4) But their significant difference decreases as λ_2 and μ_2 increases. So, β is more sensitive on EWL for lower value of λ_2 and μ_2. Because the service rate of low priority customer is greater than

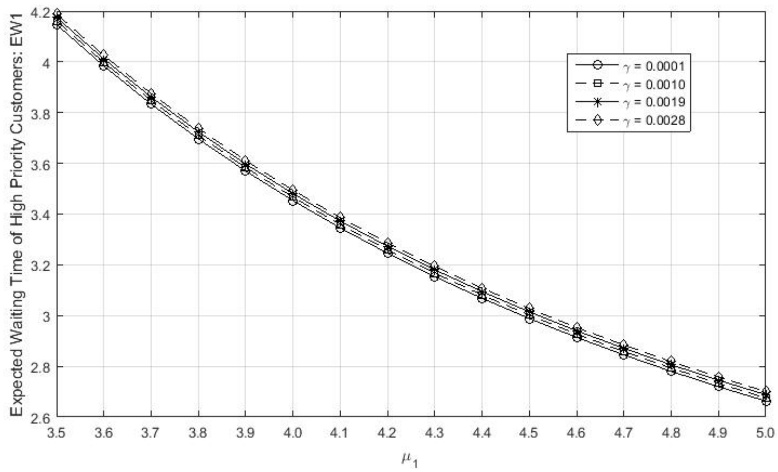

FIGURE 5. $EW1$ vs μ_1 for different values of γ

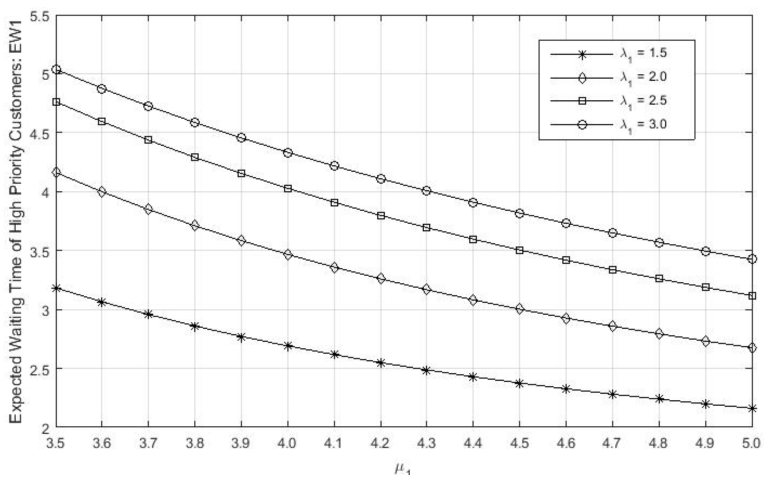

FIGURE 6. $EW1$ vs μ_1 for different values of λ_1

the arrival rate of high priority customer and the service rate of high priority customer is greater than arrival rate of low priority customer.

(5) Also, the expected waiting time of low priority customer is more sensitive when λ_2 increases and μ_2 decreases but $\lambda_2 < \mu_2$.

Example 5.3.

From Figs.13 to 18, we observe the impact of various parameters on Expected number of high priority customers lost (EL1) given below:

(1) Expected number of high priority customers lost (EL1) decreases as β and μ_1 increases but γ and λ_1 decreases.
(2) With reference of given ranges of all parameters, the rate of expected number of high priority customers lost (EL1) is minimum at the higher value of β and μ_1 but the lower value of γ as example 1. But the rate of expected number of high priority customers lost (EL1) is minimum at the lower value of λ_1. Because the expected number of high priority customer beyond the

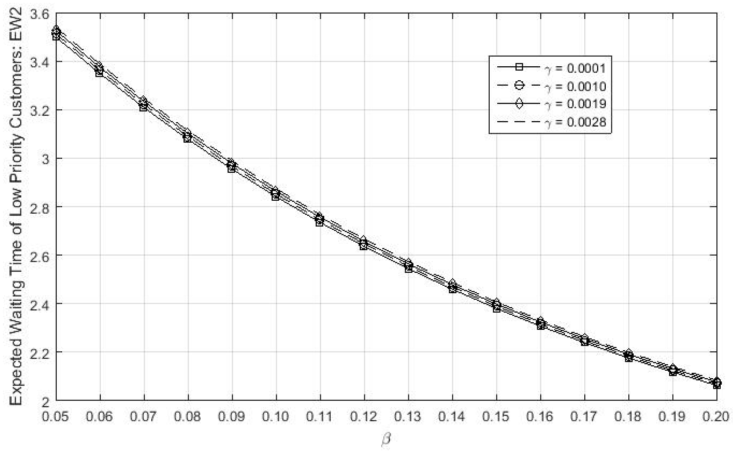

FIGURE 7. $EW2$ vs β for different values of γ

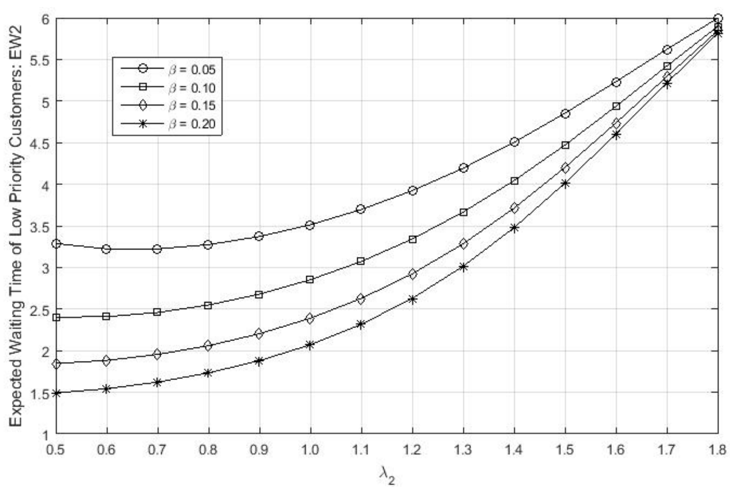

FIGURE 8. $EW2$ vs λ_2 for different values of β

threshold level doesn't come often for the lower value of λ_1 and also the service rate of low priority customer is greater than the arrival rate of high priority customer but the arrival rate of high priority customer is more than the arrival rate of low priority customer.

(3) Expected number of high priority customers lost is more sensitive when μ_1 decreases but λ_1 increases. Because the expected number of high priority customer beyond the threshold level comes often when λ_1 increases to certain specific level and also, the service rate of high priority customer is greater than the arrival rate of high priority customer.

(4) On the whole discussion β is highly significant parameter. But the comparison of other parameters γ is more effective.

Example 5.4.

From Figs.19 to 23, we observe the impact of various parameters on Expected number of low priority customers lost (EL2) given below:

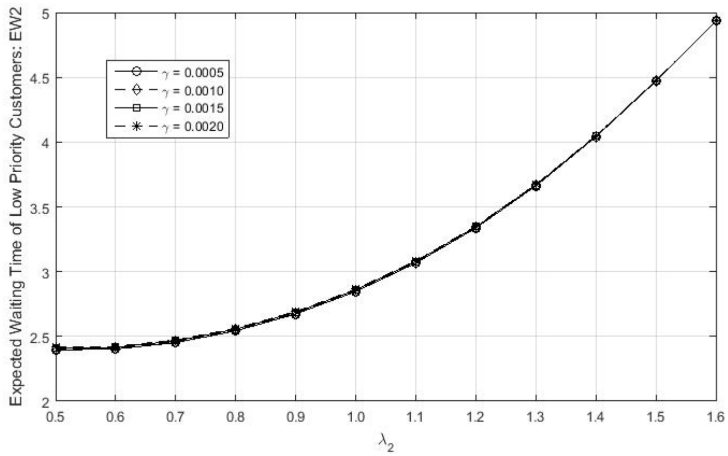

FIGURE 9. $EW2$ vs λ_2 for different values of γ

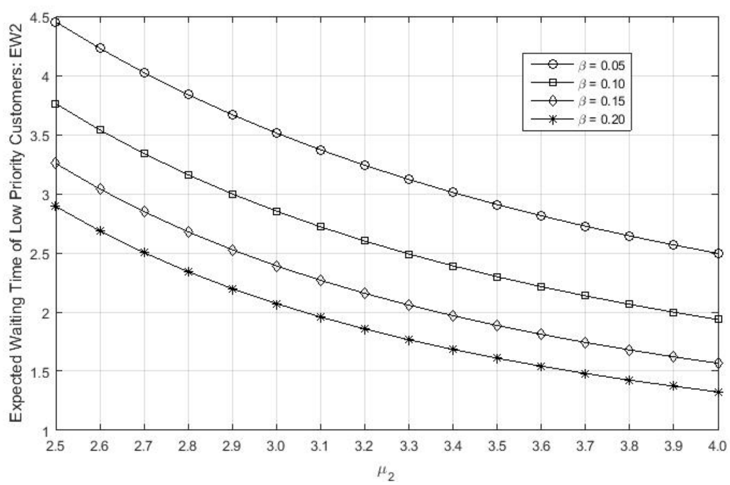

FIGURE 10. $EW2$ vs μ_2 for different values of β

(1) Expected number of low priority customers lost (EL2) decreases as β and μ_2 increases but γ and λ_2 decreases as example 5.3.
(2) β is more sensitive on EWL for a lower value of μ_2 but a moderate value of λ_2. Because the service rate of low priority customer is greater than the arrival rate of low priority customer and the arrival rate of low priority customer is lesser than the arrival rate of high priority customer.

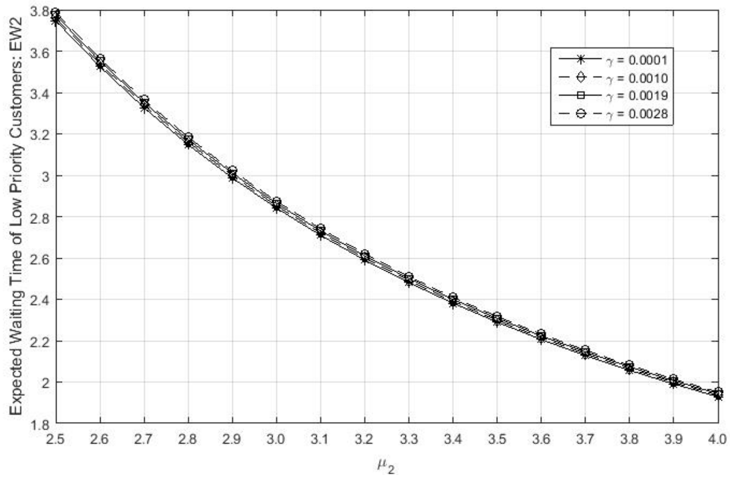

FIGURE 11. $EW2$ vs μ_2 for different values of γ

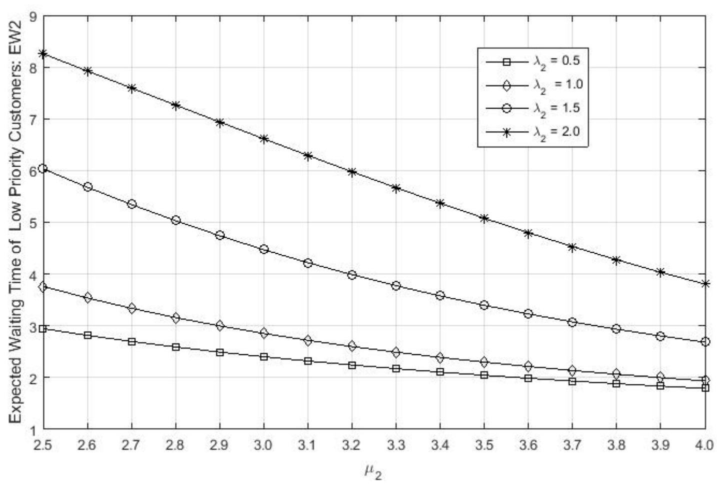

FIGURE 12. $EW2$ vs μ_2 for different values of λ_2

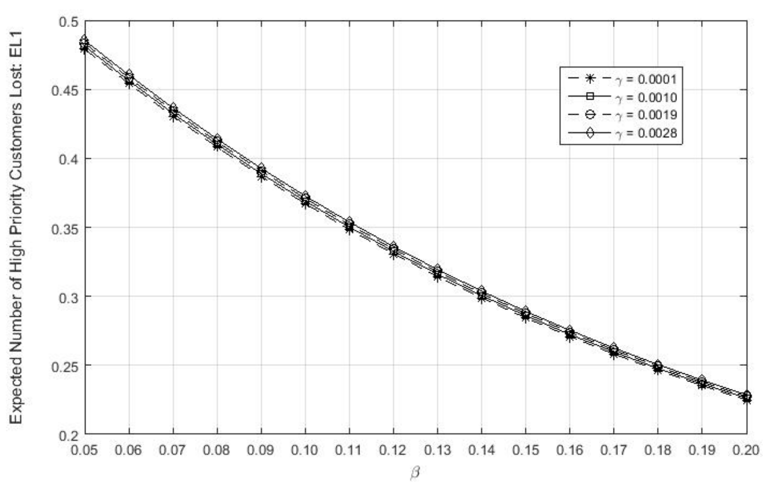

FIGURE 13. $EL1$ vs β for different values of γ

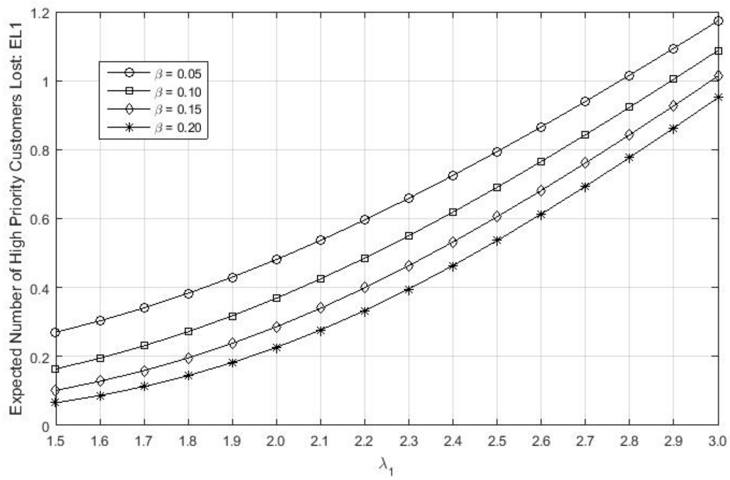

FIGURE 14. $EL1$ vs λ_1 for different values of β

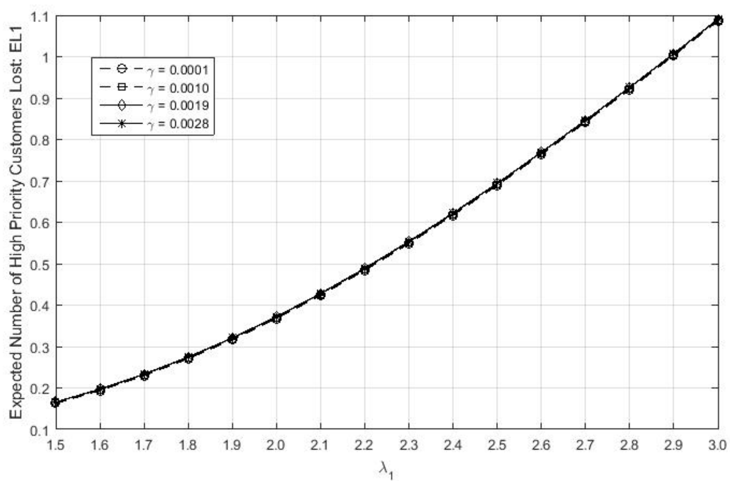

FIGURE 15. $EL1$ vs λ_1 for different values of γ

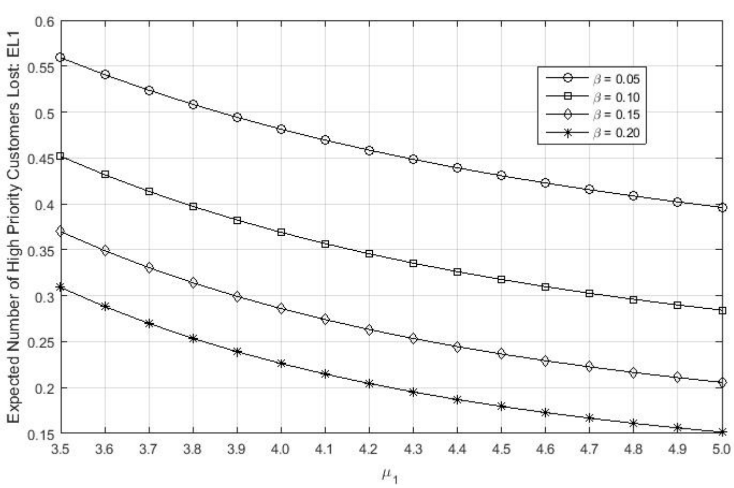

FIGURE 16. $EL1$ vs μ_1 for different values of β

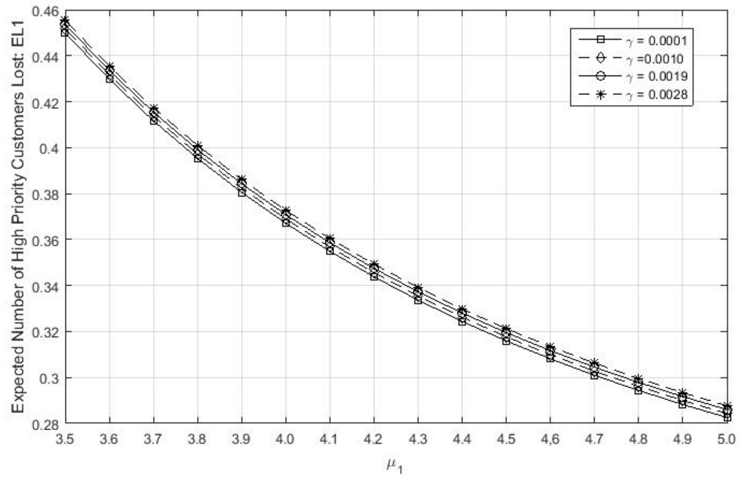

FIGURE 17. $EL1$ vs μ_1 for different values of γ

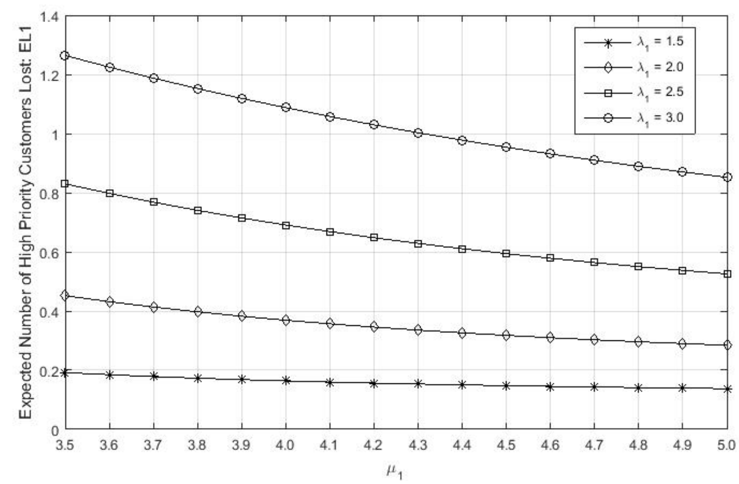

FIGURE 18. $EL1$ vs μ_1 for different values of λ_1

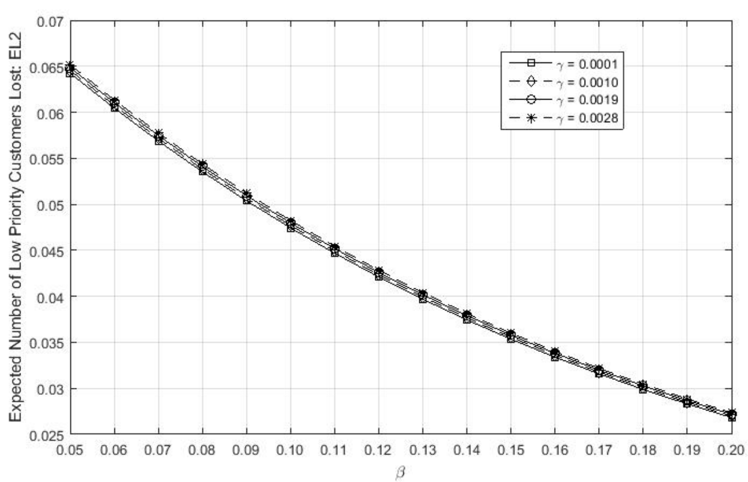

FIGURE 19. $EL2$ vs β for different values of γ

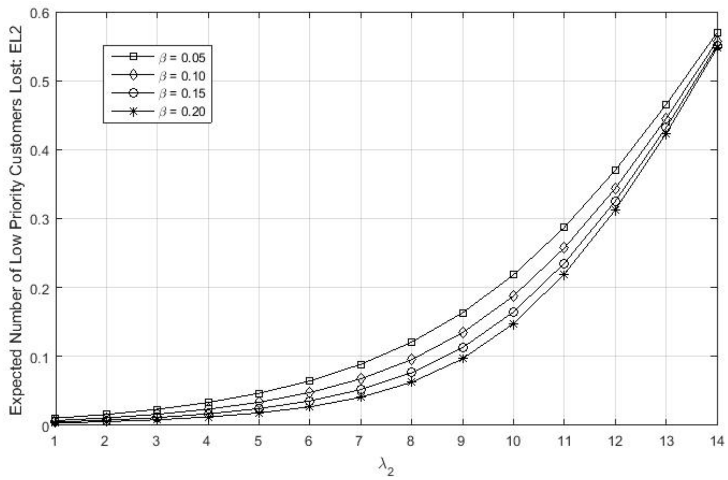

FIGURE 20. $EL2$ vs λ_2 for different values of β

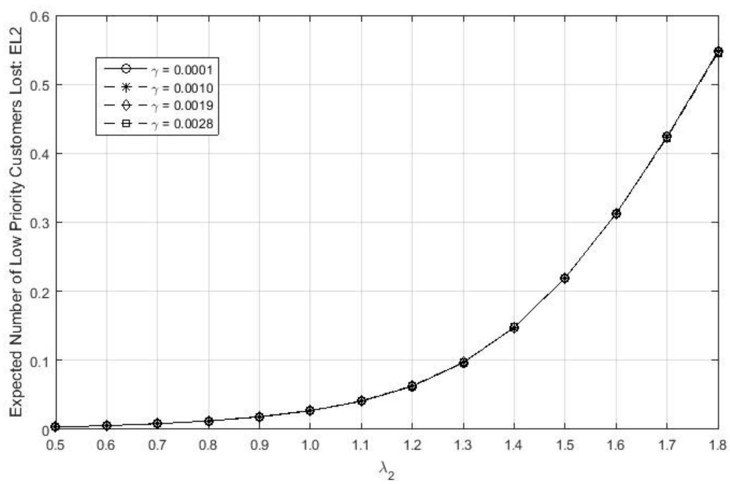

FIGURE 21. $EL2$ vs λ_2 for different values of γ

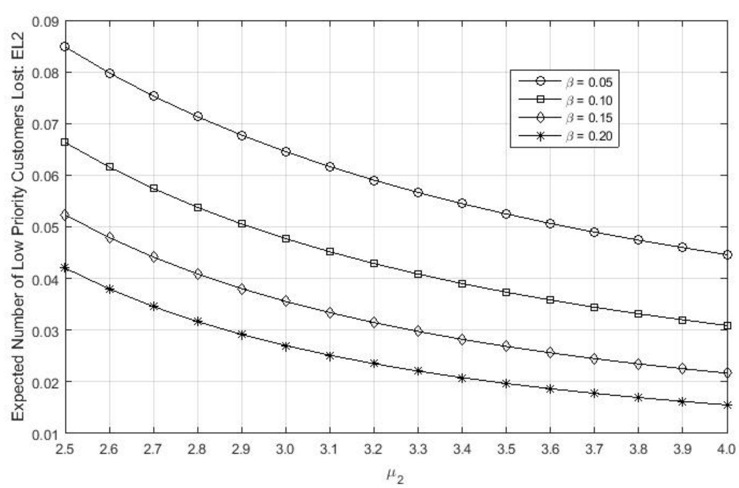

FIGURE 22. $EL2$ vs μ_2 for different values of β

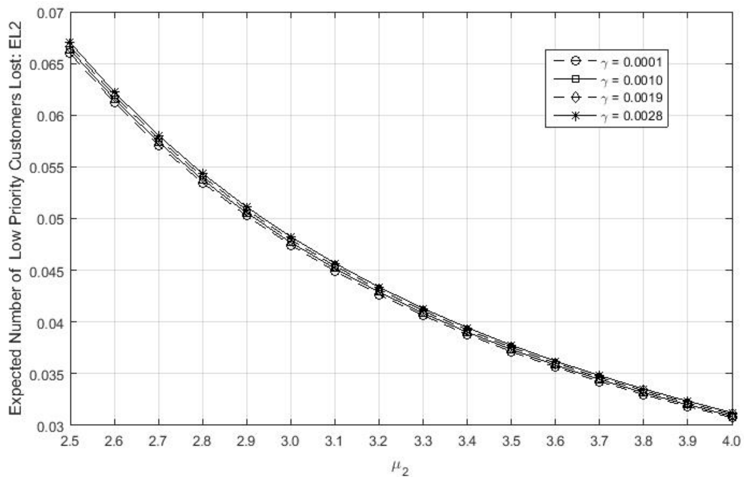

FIGURE 23. $EL2$ vs μ_2 for different values of γ

REFERENCES

[1] A.Z. Melikov, A.A., Molchanov, Stock Optimization in Transport/Storage, Cybernet. Syst. Anal. 28 (1992), 484-487.

[2] K. Sigman, D. Simchi-Levi, Light traffic heuristic for an M/G/1 queue with limited inventory, Ann. Oper. Res. 40 (1992), 371-380.

[3] O. Berman, E.H. Kaplan, D. G. Shimshak, Deterministic approximations for inventory management at service facilities, IIE Trans. 25 (1993), 98-104.

[4] O. Berman, E. Kim, Stochastic models for inventory management at service facilities, Stoch. Model. 15 (1999), 695-718.

[5] O. Berman, K.P. Sapna, Inventory management at service facilities for systems with arbitrarily distributed service times, Stoch. Model. 16 (2000), 343-360.

[6] O. Berman, E. Kim, Dynamic inventory strategies for profit maximization in a service facility with stochastic service, demand and lead time. Math. Method. Oper. Res. 60(2004), 497-521.

[7] M. Schwarz, C. Sauer, H. Daduna, R. Kulik, R. Szekli, M/M/1 queueing systems with inventory, Queueing Syst. 54 (2006), 55-78.

[8] J.R. Artalejo, A. Krishnamoorthy, M.J. Lopez-Herrero, Numerical analysis of (s,S) inventory systems with repeated attempts, Ann. Oper. Res. 141 (2006), 67-83.

[9] T.G. Deepak, A. Krishnamoorthy, V.C. Narayanan, K. Vineetha, Inventory with service time and transfer of customers and/inventory, Ann. Oper. Res. 160 (2008), 191-213.

[10] A. Z. Melikova, M. O. Shahmaliyev, Markov Models of Inventory Management Systems with a Positive Service Time, J. Comput. Sys. Sci. Int. 57 (2018), 766-783.

[11] K. Jeganathan, A. Z. Melikov, S. Padmasekaran, S. Jehoashan Kingsly, K. Prasanna Lakshmi, A Stochastic Inventory Model with Two Queues and a Flexible Server, Int. J. Appl Comput. Math. 5 (2019), 1-21.

[12] D.S. Lee, BhaskaSengupta, IEE/ACM Trans. Networking, 1 (1993), 709-717.

[13] D.I. Choi, Performance analysis of two finitebuffers queueing system with priority scheduling dependent upon queue length, J. Appl. Math. Comput. 22 (2006), 523-533.

[14] R. Chipalkatti, J.F. Kurose, D. Towsley, Scheduling policies format-time and non-rest-time traffic in a statistical multiplexer, in Proc. IEEE. (1989), 774-783.

[15] Y. Lim, J. Kobza, Analysis of a delay-dependent priority discipline in a multi-class traffic packet switching node, in Proc. IEEE, DOI: 10.1109/INFCOM.1988.13004.

[16] H. Ishi, T. Nose, Perishable inventory control with two types of customers and different selling prices under the warehouse capacity constraint, Int. J. Prod. Econ. 44 (1996), 167- 176.

[17] N. Zhao, Z. Lian, A queueing-inventory system with two classes of customers, Int. J. Prod. Econ. 129 (2011), 225-231.

[18] B. Sivakumar, G. Arivarignan, A modified lost sales inventory systemwith two types of customers, Qual. Technol. Quant. M. 5 (2008), 339-349.

[19] K. Jeganathan, N. Anbazhagan, J. Kathiresan, A Retrial inventory system with non-preemptive priority service, Int. J. Info. Management Sci. 24 (2013), 57-77.

[20] K. P. SapnaIsotupa, S. K. Samanta, A continuous review (s,Q) inventory system with priority customers and arbitrarily distributed lead times, Math. Comput. Model. 57 (2013), 1259-1269.

[21] T. Karthick, B. Sivakumar, G. Arivarignan, An inventory system with two types of customers and retrial demands, Int. J. Syst. Sci. 2 (2015), 90-112.

[22] K. Jeganathan, J. Kathiresan, N. Anbazhagan, A retrial inventory system with priority customers and second optional service, OPSEARCH, 53 (2016), 808-834.

[23] A. Z. Melikova, A. Krishnamoorthy, M. O. Shahmaliyev, Numerical Analysis and long run total cost optimization of perishable queueing inventory systems with delayed feedback, Queueing Models and Service Management, 2 (2019), 83-111.

[24] M. Amirthakodi, B. Sivakumar, An inventory system with service facility and finite orbit size for feedback customers, Opsearch, 52 (2015), 225-255.

[25] M. Amirthakodi, V. Radhamani, B. Sivakumar, A perishable inventory system with service facility and feedback customers, Ann. Oper. Res. 233 (2015), 25-55.

[26] C. Suganya, M. Amirthakodi, B. Sivakumar, An inventory system with two heterogeneous servers and orbital search for feedback customers, Int. J. Math. Model. Numer. Optim. 8 (2018), 351-377.