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EXTENSION OF LINMAP MODEL TO MINKOWSKI DISTANCE METRIC OF ORDER 3 FOR THE OPTIMAL ESTIMATION OF INDIVIDUAL UTILITY FUNCTION

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Abstract. This paper extends the conventional LINMAP model to the Minkowski distance metric of order 3 for the simultaneous determination of the ideal points and weights for the optimal estimation of utility functions by Decision Makers. The algorithm for solution of the extended LINMAP model is developed, and numerical examples are presented to demonstrate the applicability of the extended model. The result shows that Minkowski distance metric gives a more accurate and distinctive ranking of alternatives than the Euclidean distance metric.

Keywords: LINMAP; Minkowski metric; Utility function; Ideal point; Weight.

2010 AMS Subject Classification: 65Y20, 90B50.

1. Introduction

Decision making is part of our daily lives and optimal estimation of utility functions involves the use of certain multiple criteria. Multi-Criteria Decision Making (MCDM) problems according to (Zeinab et al., 2010) citing Hwang et al. (1981) are categorized into Multi-Attribute Decision Making (MADM) and Multi-Objective Decision Making (MODM). In the optimization problems of the MODM, several objective functions should be satisfied and the decision

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space is continuous while MADM associates itself with the problems in which the set of decision alternatives have been predetermined (i.e. concentrates on problems with discrete decision spaces (Vazifedost et al.,2011)). This invariably involves making preference decisions such as evaluation, prioritization and selection over available finite number of alternatives that are characterized by multiple, and usually, conflicting attributes (Yoon et al., 1995). Therefore, MADM is one of the widely used decision methodology for most real world decision making problems (Tuli et al., 2011).

MADM problems can be divided into different categories depending on the criteria defined. The DM can be divided into different categories depending on the criteria defined. The DM can be consulted or the final decision can be made solely based on some existing data. (Triantaphyllou, 2000) opined that the availability of a wide selection of methods for solving MADM problems generates the paradox that the selection of a MADM method for a given problem can be lead to another MADM problem itself which according to (Yeh, 2003) is caused by inconsistent ranking problem. (Jahanshaloo et al., 2011) stated that the choice of a specific method in general influences the ranking outcome.

Mathematical programming models have been used in a number of ways to obtain optimal estimate of utility functions of decision makers (DM). Several existing methods have been used to deal with these kind of problems, these include TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) developed by (Hwang and Yoon, 1981), the Multi-Dimensional Scaling method (MDS) with ideal point (Kruskaal, 1964a, 1964b), the Total Sum (TS), the Weighted Product Model (WPM), the Outranking approaches Elimination and Choice Translation Reality, ELECTRE (Benayoun et al., 1966) and PROMETHEE (Brans and Vincke, 1985), LINear programming technique for Multidimensional Analysis of Preferences, (LINMAP) (S-rinivasan and Shocker, 1973a), interactive Simple Additive Weighting (SAW) method (Hwang and Yoon, 1981), artificial neural network method (Malakooti and Zhou, 1994), non-linear programming method based on fuzzy preference relation (Fan et al., 2002), interval numbers based optimization approach (Xu, 2004).

Some applications in practice are prevented because of the complexities of most of these methods as perceived by real DMs. TOPSIS and LINMAP are two well-known MADM methods though they require different types of information (Deng-Feng, 2008). In the TOPSIS method, the decision matrix, V, and the weight vector, w, are given as crisp values a priori. A Positive Ideal Solution (PIS) and a Negative Ideal Solution (NIS) are generated from V directly. The best compromise alternative is then defined as the one that has the shortest distance to the PIS and the farthest from the NIS. TOPSIS still suffers from ranking abnormalities caused by the weighting algorithm used to weigh different criteria, (Lahby et al., 2012a, 2012b).

However, in the LINMAP method, the weight vector, w, and the PIS are unknown a priori. It is based on pair-wise comparisons of alternatives given by the DM and generates the best compromise alternatives as the solution that has the shortest distance to the PIS (Xia et al., 2006). (Xia et al.,2006) extended the conventional LINMAP to solve group MADM problems with fuzzy information using triangular fuzzy numbers to assess alternatives with respect to qualitative attributes in their fuzzy linear programming model. A fuzzy LINMAP method to handle group decision making problems involving linguistic variables and incomplete preference information was developed by (Li and Xia, 2007) to capture the uncertainty in the DM's preferences and to calculate the distances between the alternatives and the ideal point. Another LINMAP method in group decision making environment was proposed by (Li, 2008) by transforming the problem as a possibility programming with multiple objectives.

The LINMAP, as proposed by (Srinivasan and Shocker, 1973b) for the optimal estimation of utility functions of DM, dealt extensively with the preference to those stimuli which are closer to DM's ideal point in terms of a weighted Euclidean distance measure by proposing a linear programming model for external analysis (which is the estimation of the coordinates of his ideal point and the weights) pre specified by their coordinate locations in the multidimensional space.

2. Statement of the problem

A Linear Programming (LP) model for the estimation of individual utility function (i.e. estimation of the coordinates of ideal point and weights) using the Euclidean distance measure formulated by (Srinivasan and Shocker, 1973b) was extended to general Minkowski metric with the assumption that the ideal point locations are already known and thereby only estimating the weights. The methodology was equally extended to the city block metric for joint determination of weights and ideal points; they could not extend the methodology to the general Minkowski metric for the simultaneous determination of the ideal points and the weights. This problem remains unsolved till date.

This paper shows how LINMAP model can be extended to the general Minkowski metric of order 3 for the simultaneous determination of the ideal point and weights, and how it can be used in solving MADM problems in real life.

3. Methodology

3.1 Steps for solving MADM problems

There are three steps involved in utilizing any decision making technique involving numerical analysis of alternatives. They are:

i. Determining the relevant criteria and alternatives;

ii. Attaching numerical measures to the relevant importance of the criteria and to the impacts of the alternatives on these criteria;

iii. Processing the numerical values to determine the ranking of each alternative.

3.2 The Decision Matrix

The decision matrix, V (or the performance table), is an array presenting on one axis a list of alternatives that are evaluated regarding on the other axis, a list of criteria, which are weighted dependently of their respective importance in the final decision to be taken. It has as its elements (V_{ij}). These are values allocated to the i^{th} alternative of j^{th} criteria. Suppose the DM has to choose one or rank m possible alternatives, A_i (i = 1,2,...,m) based on n attributes, C_j $(j = 1, 2, ..., n)$. Each row (line) of the matrix expresses the performances of alternatives i relative to the n attributes considered. Each column j expresses the evaluations of all the alternatives adopted by the DM relative to the alternatives. To represent the importance of the attributes, a weight w_j for each criterion can be given resulting in a vector, $W = (w_1, w_2, ..., w_n)$. In some cases the weights sum to one (i.e $\sum_{j=1}^{n} w_j = 1$) implying that each weight can be interpreted as the percentage of importance of the corresponding attributes and $w_j \geq 0 (j = 1, 2, ..., n)$. Hence, a multi-attribute decision making problem can be concisely expressed in matrix format as follows:

3.3 Linear programming technique for multi dimensional analysis of preference (LINMAP):

Considering a set of feasible m alternatives $A = \{A_1, A_2, ..., A_m\}$ consisting of a collection of n attributes or criteria $C = \{C_1, C_2, ..., C_n\}$ with which the performance of the alternatives are measured or on which the DM makes his preferences judgments. To carry out the analysis for each DM separately, since the analysis does not involve any comparisons across DMs, the DMs' ideal points and weights based on their preference judgments on a set of alternatives whose locations are pre-specified in a multidimensional space are determined. Let the ideal point $Y = y_p,\ p \in P$. This ideal point yp could be positive, negative or zero. Once the ideal point is identified, the alternative with the shortest distance from the ideal point is then selected as the "most preferred" (Shadi-Nezhad and Akhtari, 2008).

3.4 The Euclidean metric distance

Let V_i denote the values allocated to i^{th} attribute in the t-dimensional space such that $V_i = \{v_{ip}\}\ p \in P$. The un-weighted Euclidean metric distance, d_i^u, of the preferred alternative from the ideal point is given by

$$d_i^u = [\sum_{p \in P} (v_{ip} - y_p)^2]^{\frac{1}{2}}. \tag{1}$$

Let $\Omega = \{(j,k)\}$ denote the set of ordered pairs (j,k) where j represents the preferred alternative on a forced choice basis (the DM chooses each alternative from among the remaining alternatives without restoration) resulting from a pair-wise comparison involving j and k. Ω normally, but not necessarily, has $\frac{m(m-1)}{2}$ elements and for every ordered pair $(j,k) \in \Omega$, the alternative A_k is closer to the positive ideal solution than the alternative A_j if $d_k^u \geq d_j^u$ or $d_j^u \leq d_k^U$. Let $W = \{w_p\},\ p \in P$ denote the weights associated with the t-dimensions, then to represent the importance of each attribute, a relative weight wp of the i^{th} criteria can be given which are considered in order to change the present measure into the same measures. Hence, the weighted

Euclidean metric distance, d_i, of the alternatives from the ideal point is given by

$$d_i = [\sum_{p \in P} W_p (V_{ip} - Y_p)^2]^{\frac{1}{2}}. \tag{2}$$

Therefore, the squared weighted distance $s_i = d_i^2$ is given by

$$s_i = \sum_{p \in P} W_p (V_{ip} - Y_p)^2 \tag{3}$$

The performance ratings and the attribute weights are cardinal values that represent the DM's absolute preference (Yeh, 2003). In many cases the weights sum up to one i.e. $\sum_{p \in P} = 1$ for every $p \in P$, so that each weight can be interpreted as the percentage of importance of corresponding attribute. This weighted distance implies that the closer the alternative is to the ideal point, the more that alternative is preferred. From this implication, if the deviation from the ideal point is much, then s_k is increased. For this property to apply, it is assumed that the attribute weight is constrained to be non-negative, i.e. $W_p \geq 0$ for $p \in P$. The following constrained optimization can, therefore, be solved to obtain (w_p, y_p):

$$MinH = \sum_{(j,k) \in \Omega} Z_{jk} \tag{4}$$

s.t. $(s_k - s_j) + z_{jk} \geq 0, (j,k) \in \Omega$ $\sum_{(j,k) \in \Omega} (s_k - s_j) = g$, $Z_{jk} \geq 0, (j,k) \in \Omega$. Using suitable transformations, (20) therefore, reduces to the LP formulation thus,

$$Min \sum_{(j,k) \in \Omega} Z_{jk} = Z \tag{5}$$

Subject to:

$\sum W_p a_{jkp} + \sum q_p b_{jkp} + Z_{jk} \geq 0$

$\sum W_p A_p + \sum q_p D_p = g$

$W_p \geq 0$

q_p urs

$Z_{jk} \geq 0$ for $(j,k) \in \Omega$, $p \in P$.

4. Extension of LINMAP to Minkowski metric of order 3

To simultaneously determine the ideal points locations $\{y_p\}$ and the weights $\{w_p\}$ when the pre-specified locations of the i^{th} attribute in the t-dimensional space $V = \{v_{ip}\}$, $p \in P$ are given, the weighted Minkowski metric distance, d_i, of the i^{th} attribute from the ideal point is given by

$$d_i = [\sum_{p\in P} W_p |v_{ip} - y_p|^f]^{\frac{1}{f}}, f \geq 1, \forall i \in \Omega. \tag{6}$$

For computational convenience we define d_i as follows

$$d_i = [\sum_{p\in P} W_p (v_{ip} - y_p)^f]^{\frac{1}{f}}, \forall i \in \Omega. \tag{7}$$

For $f = 3$, we have

$$d_i = [\sum_{p\in P} W_p (v_{ip} - y_p)^3]^{\frac{1}{3}}, \forall i \in \Omega, \tag{8}$$

$$d_i^3 = \sum_{p\in P} W_p (v_{ip} - y_p)^3, \tag{9}$$

$$d_i^3 = \sum_{p\in P} W_p (v_{ip}^3 - 3y_p v_{ip}^2 + 3y_p^2 v_{ip} - y_p^3), \tag{10}$$

$d_i^3 = \sum_{p\in P} W_p V_{ip}^3 - 3\sum_{p\in P} W_p Y_p v_{ip}^2 + 3\sum_{p\in P} W_p Y_p^2 V_{ip} - \sum_{p\in P} W_p Y_p^3$. Defining, $s_i = d_i^3$, then

$$s_i = \sum_{p\in P} W_p V_{ip}^3 - 3\sum_{p\in P} W_p Y_p V_{ip}^2 + 3\sum_{p\in P} W_p Y_p^2 V_{ip} - \sum_{p\in P} W_p Y_p^3. \tag{11}$$

It is important to note that s_i may not be nonnegative, however, it still follows if $S_i < 0$, since the model merely states that j will be preferred to k if $S_k > S_j$ and does not require S_i to be positive. Hence, the attribute weight could be constrained to be non-negative i.e. $W_p \geq 0$ for $p \in P$. Therefore for the pair $(j,k) \in \Omega$, $s_j = \sum_{p\in P} w_p v_{jp}^3 - 3\sum_{p\in P} w_p y_p v_{jp}^2 + \sum_{p\in P} w_p y_p^2 v_{jp} - \sum_{p\in P} w_p y_p^3$, $s_k = \sum_{p\in P} w_p v_{kp}^3 - 3\sum_{p\in P} w_p y_p v_{kp}^2 + 3\sum_{p\in P} w_p y_p^2 v_{kp} - \sum_{p\in P} w_p y_p^3$,

$$s_k - s_j = \sum_{p\in P} w_p (v_{kp}^3 - v_{jp}^3 - 3\sum_{p\in P} w_p y_p (v_{kp}^2 - v_j p^2) + \sum_{p\in P} w_p y_p^2 (v_{kp} - v_{jp}). \tag{12}$$

Let $q_p = w_p y_p$ and $r_p = w_p y_p^2$. Then $r_p = w_p \frac{q_p^2}{w_p^2} = \frac{q_p^2}{w_p}$. For $w_p > 0$ and q_p, that is, $q_p = \sqrt{w_p r_p} \leq \frac{w_p + r_p}{2}$. This implies that $2q_p - w_p - r_p \leq 0$, for all $w_p \geq 0$; $p \in P$, $s_k - s_j = \sum_{p\in P} w_p (v_{kp}^3 - v_{jp}^3) - 3\sum_{p\in P} w_p (v_{kp}^2 - v_{jp}^2) + 3\sum_{p\in P} r_p (v_{kp} - v_{jp})$ for all real values v_{jp}, v_{kp}; $w_p \geq 0$; q_p unrestricted; $s_k - s_j \geq 0$, $(j,k) \in \Omega$ and $p \in P$. Defining

$$a_{jkp} = (v_{kp}^3 - v_{jp}^3), \tag{13}$$

(14) $$b_{jkp} = -3(v_{kp}^2 - v_{jp}^2),$$

(15) $$c_{jkp} = 3(v_{kp} - v_{jp}),$$

for $(j,k) \in \Omega$ and $p \in P$. We have

(16) $$s_k - s_j = \sum_{p \in P} w_p a_{jkp} + \sum_{p \in P} q_p b_{jkp} + \sum_{p \in P} \gamma_p c_{jkp}).$$

Then $g = \sum_{(j,k) \in \Omega}(s_k - s_j)$ can be written as

(17) $$g = \sum_{p \in P}(\sum_{p \in P} w_p a_{jkp} + \sum_{p \in P} q_p b_{jkp} + \sum_{p \in P} \gamma_p c_{jkp}).$$

Let

(18) $$A_p = \sum_{p \in P} a_{jkp},$$

(19) $$B_p = \sum_{p \in P} b_{jkp},$$

(20) $$C_p = \sum_{p \in P} c_{jkp}.$$

Equation (17) becomes

(21) $$g = \sum_{p \in P} w_p A_p + \sum_{p \in P} q_p B_p + \sum_{p \in P} \gamma_p C_p = 1.$$

The model formulation now becomes

(22) $$Min Z = \sum_{(j,k) \in \Omega} Z_{jk}$$

subject to:

$\sum_{p \in P} w_p a_{jkp} + \sum_{p \in P} q_p b_{jkp} + \sum_{p \in P} \gamma_p c_{jkp} + z_{jk} \geq 0$

$\sum_{p \in P} w_p A_p + \sum_{p \in P} q_p B_p + \sum_{p \in P} \gamma_p C_p = 1$

$2q_p - w_p - \gamma_p \leq 0$; $p \in P$

$z_{jk} \geq 0$; $(j,k) \in \Omega$

$w_p \geq 0$; $p \in P$

$\gamma_p \geq 0$; $p \in P$

q_p unrestricted in sign; $p \in P$

5. The E-linmap algorithm

To estimate the weights $\{w_p^*\}$ and the ideal point $\{y_p^*\}$.

Step 1. Let P denote the set of n attributes $\{1,2,...,n\}$ and J denote the given set of m alternatives $\{1,2,...,m\}$. Let $V=\{v_{ip}\}$ denote the given attribute values for the alternative (v_{ip} is the value of the i^{th} alternative on the p^{th} attribute). Let $\Omega=\{(j,k)\}$ denote the given set of ordered pairs (j,k) such that k is preferred to j on a forced choice basis in the comparison involving j and k.

Step 2. Compute a_{jkp} and b_{jkp} for each pair $(j,k)\in\Omega$ and for every attribute $p\in P$. Compute A_p and B_P and C_p.

Step 3. Solve the E - LINMAP model.

Step 4. Compute the index of fit $C^* = H*/1+H^*$.

Step 5. Compute the distance measures and rank the alternatives.

Note that, for $p\in P$

a. if $w_p^*>0$ then $y_p^*=q_p^*/w_p^*$,

b. if $w_p^*=0$ and $q_p^*=0$ define $y_p^*=o$,

c. if $w_p^*=0$ and $q_p^*=0$ define $y_p^*>o$ then $y_p^*=+\infty$,

d. $w_p^*=0$ and $q_p^*<0$ define $y_p^*>o$ then $y_p^*=-\infty$.

6. Numerical applications

6.1. Using Euclidean distance metrics

The data consist of five stimuli $(n=5)$ in a two dimensional space $(t=2)$ with coordinates $V_1=(0,5), V_2=(5,4), V_3=(0,2), V_4=(1,3), {}_5=(4,1)$ The forced choice ordered paired comparison judgments are: $\Omega=\{(1,2),(3,1),(4,1),(5,1),(2,3),(2,4),(2,5),(4,3),(3,5),(4,5)\}$ The linear program formulation to obtain the 'best' weights and ideal point, is given below:

$$MinZ = Z_{12}+Z_{31}+Z_{41}+Z_{51}+Z_{23}+Z_{24}+Z_{25}+Z_{43}+Z_{35}+Z_{45} \tag{23}$$

subject to

$25-9w_2-10q_1+2q_2+z_{12}\geq 0, 0+21w_2-6q_2+z_{31}\geq 0, -w_1+16w_2+2q_1-4q_2+z_{41}\geq 0,$

$-16w_1+24w_2+8q_1-8q_2+z_{51}\geq 0$, $-25w_1-12w_2+10q_1+4q_2+z_{23}\geq 0$, $-24w_1-7w_2+8q_1+2q_2+z_{24}\geq 0$, $-9w_1-15w_2+2q_1+6q_2+z_{25}\geq 0$, $-w_1-5w_2+2q_1+2q_2+z_{43}\geq 0$, $16w_1-3w_2-8q_1+2q_2+Z_{35}\geq 0$, $15w_1-8w_2-6q_1+4q_2\geq z_{45}$, $-20w_1+2w_2+8q_1+4q_2=1$, $w_1,w_2,z_{12},z_{31},z_{41},z_{51},z_{23},z_{24},z_{25},z_{43},z_{35},z_{45}\geq 0$, q_1,q_2 are unrestricted in sign.

Using Microsoft Excel Solver software in HP-G56 with processor: Pentium (R) Dual-Core CPU T4500 @ 2.30GHZ of 4.00GB memory size to solve the LP (61) the optimal solution is found to be: $w_1=0.0278$, $w_2=0.0556$, $q_1=0.0833$, $q_2=0.1944$ $z_{12}=0.25$, $z_{31}=z_{41}=z_{51}=z_{23}=z_{24}=z_{25}=z_{43}=z_{35}=z_{45}=0$, $z=0.25$ From Equation, $q_p=w_p y_p$, for $p\in P$ it follows that the ideal points y_p is calculated as $y_1=3$ and $y_2=3.5$ Using $Y=3,3.5$ and $W'=0.0278,0.0556$, the distance measures are $s_1=0.375$, $s_2=0.125$, $s_3=0.375$, $s_4=0.125$, $s_5=0.375$ $s_1=s_3=s_5>s_2=s_4$.

6.2. Numerical applications using the Minkoski metric of order 3

Using the same values as in Example (1) above The data consist of five stimuli $(n=5)$ in a two dimensional space $(t=2)$ with coordinates $V_1=(0,5)$, $V_2=(5,4)$, $V_3=(0,2)$, $V_4=(1,3)$, $V_5=(4,1)$ The forced choice ordered paired comparison judgments by the DM are: $\Omega=\{(1,2),(3,1),(4,1),(5,1),(2,3),(2,4),(2,5),(4,3),(3,5),(4,5)\}$ The LP formulation is $MaxZ=Z_{12}+Z_{31}+Z_{41}+Z_{51}+Z_{23}+Z_{24}+Z_{25}+Z_{43}+Z_{35}+Z_{45}$, subject to: $125w_1-61w_2-75q_1+27q_2+15r_1-3r_2+z_{12}\geq 0$, $117w_2-63q_2+9r_2+z_{31}\geq 0$, $-w_1+98w_2+3q_1-48q_2-3r_1+6r_2+z_{41}\geq 0$, $64w_1-7w_2-48q_1+9q_2+12r_1-3r_2+z_{35}\geq 0$, $63w_1-26w_2-45q_1+24q_2-9r_1-6r_2+z_{45}\geq 0$, $-124w_1+70w_2+60q_1-6q_2-12r_1-6r_2=1$, $w_1-2q_1+r_1\geq 0$, $w_2-2q_2+r_2\geq 0$, $w_1,w_2,r_1,r_2,z_{12},z_{31},z_{41},z_{51},z_{23},z_{24},z_{25},z_{43},z_{35},z_{45}\geq 0$, q_1,q_2 unrestricted in sign.

Using Microsoft Excel Solver software in HP-G56 with processor: Pentium (R) Dual-Core CPU T4500 @ 2.30GHZ of 4.00GB memory size to solve the LP (62), the optimal solution is found to be $w_1=0.013158$, $w_2=0.022281$, $q_1=0.022036$, $q_2=0.041429$, $r_1=0$, $r_2=0.000349$ $z_{12}=0.25$, $z_{31}=z_{41}=z_{51}=z_{23}=z_{24}=z_{25}=z_{43}=z_{35}=z_{45}=0$, $z=0.25$ Consequently, the Index of Fit $C^*=H^*/1+H^*=0.25/1+0.25=0.2$ Therefore, the LP has a finite optimum solution since $z_{12}=0.25>0$. With this finite optimum, the ideal point locations are given as $y_1 = \frac{q_1}{w_1} = \frac{0.022036}{0.013155} = 1.68$, $y_2 = \frac{q_2}{w_2} = \frac{0.041429}{0.022281} = 1.86$. Computing the distances with the optimal values of the LP yields $(0,5) = V_1 : S_1 = 0.013155(0-1.68)3 + 0.022281(5-1.86)^3 = 0.6274$,

$(5,4) = V_2 : S_2 = 0.013155(5-1.68)^3 + 0.022281(4-1.86)^3 = 0.6998$,

$(0,2) = V_3 : S_3 = 0.013155(0-1.68)^3 + 0.022281(2-1.86)^3 = -0.062$,

$(1,3) = V_4 : S_4 = 0.013155(1-1.68)^3 + 0.022281(3-1.86)^3 = 0.0289$,

$(4,1) = V_5 : S_5 = 0.013155(4-1.68)^3 + 0.022281(1-1.86)^3 = 0.1501$.

Thus the ranking of the alternatives based on the Minkoski distance metric of order 3 is $S_2 > S_1 > S_5 > S_4 > S_3$.

7. Conclusion

Using the Minkowski distance metric of order 3, strict ordering of preference is obtained. It shows that, in our example, the third item is the most preferred, followed by the fourth, fifth, first and second. While the Euclidean distance metric lead to ties in choice preference. The most preferred in this case being either the second or the fourth item followed by either the first or third or fifth item. Thus, Minkowski metric of order 3 and in fact higher order has the potential to provide a more accurate estimate of the utility function than the Euclidean metric.

Conflict of Interests

The authors declare that there is no conflict of interests.

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