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OPTIMAL CONTROL OF RIFT VALLEY FEVER UNDER BUDGET CONSTRAINTS

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Abstract. An SIR-type optimal control model of Rift Valley Fever under budget constraints have been established to evaluate the potential preference in applying intervention between infected livestock and humans. The results show that resources should be allocated first for interventions focusing on population with lower level of infected individuals which are humans and thereafter shift to population with high level of infected individuals which are the livestock. Therefore, the optimal solution is a switching strategy in which priority is given to population with less infected individuals before switching to population with more infected individuals.

Keywords: RVF; optimal control; interior solution; marginal value; shadow price; limited resource.

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1. Introduction

The control of disease normally involves the utilisation of resources such as manpower, equipments, drugs and other resources which may sometimes be limited. The nature of the disease dynamics can also influence the utilisation of these resources. When the disease infects more than one host population as in Rift Valley Fever (RVF) [1-2] proper utilisation of resources

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to control the disease is difficult compared to case when the disease infect only one host population. Also, targeting control of infection on one host population for the case of multi-host transmission may affect the potential for the disease spread to another population and hence the control may even become difficult. However, when there are sufficient resources, there are possibilities to control the infection in any given mode of the disease propagation. The problem arises when we have limited resources or we have budget constraints, that we can not attend all infected individuals.

The study of optimal control in epidemiological models incorporating economic theory have been of much interest for informed decision-making. The study by Ndeffo Mbah and Gilligan [3] addressed the problem of resource allocation for disease management in multiple regions using a combination of optimisation methods for economics theory of disease control with metapopulations models from epidemiological theory. The study showed that when faced with the dilemma of choosing between socially equitable and purely efficient strategy, the choice of the control strategy should be informed by key measurable epidemiological factors such as the basic reproduction number and the efficiency of the treatment model.

The study on optimal control of epidemics in metapopulations by Rowthorn *et al.* [4] showed that rather than targeting the region with most infecteds, as might have been intuitively expected, it is instead optimal to give preference to treating the region with the lower level of infecteds. The remaining regions are treated as residual claimants receiving treatment only when there is resource left over. In this case, preferential treatment in a region with low level of infection is equivalent to giving preference to the region with the highest level of susceptible available for infection. The intuitive interpretation is that on average, an infected individual infects more than one susceptibles, so removing infecteds where susceptible are plentiful reduces the force of infection and so it is likely to bring the epidemics under control.

Ndeffo Mbah and Gilligan [6] also studied the optimal control strategies for epidemics in heterogeneous population with symmetric and asymmetric transmission. They analyse the optimal control problem to optimise the control of a pathogen that is capable of infecting multiple host with different rates of transmission within and between species. Their model consisted of two classes of host-pathogen system, comprising two host species and a common pathogen, one with asymmetric and the other with symmetric transmission rates applicable to a wide range of SI-epidemics of plant and animal pathogens. It was shown that for the asymmetric case, it is optimal to give priority in treating disease to the more infectious species, and to treat the other species only when there are resources left over. For the symmetric case, it was shown that although a switching strategy is an optimum, in which preference is first given to the species with lower level of susceptibility and then to the species with higher level of susceptibility, a simpler strategy that favours treatment of infected hosts for the more susceptible species is a robust alternative for practical application when the optimal switching time is unknown.

Other studies coupling epidemiological and economical models in order to identify optimal strategies for disease control include [6-11]. In this paper, we explore how best we can utilise resources for RVF control when we have budget constraint.

2. Materials and methods

2.1 Discounting rate and current value Hamiltonian

In order to determine the optimal trade-off between current and future revenue in intervention, it is important to take into account the current value of all future costs using the discounting rate ρ and assume that this discount rate is compounded annually. The discount rate ρ is interpreted as that: the current value of an amount which would have a value of 1 at time t years in the future is $(1-\rho)^t$ [12]. Furthermore, if the discount is compounded n times per year, then the discount rate is ρ/n per discount period, and in t years, there would be nt discount periods. The current value of an amount 1 at t years in the future will therefore be $(1-\rho/n)^{nt}$, and the current value under continuous compounding would be equal be equal to limit of $(1-\rho/n)^{nt}$ as t becomes large [12]. Since

$$\lim_{n\to\infty}(1-\rho/n)^{nt} = e^{-\rho t},$$

then any objective functional can be discounted by a factor $e^{-\rho t}$.

For many problems of economic interests, future value of revenue and expenditures are discounted [12-17] and the optimal problem is

$$\max \int_0^T e^{-\rho t} f(t,x(t),u(t))dt, \tag{1}$$

subject to

$$x'(t) = g(t, x(t), u(t)), x(0) = x_0, \tag{2}$$

and the Hamiltonian

$$H = e^{-\rho t} f(t, x(t), u(t)) + \lambda(t) g(t, x(t), u(t)) \tag{3}$$

such that $(x(t), u(t), \lambda(t))$ satisfy

$$\frac{\partial H}{\partial u} = e^{-\rho t} \frac{\partial f(t, x(t), u(t))}{\partial u} + \lambda(t) \frac{\partial g(t, x(t), u(t))}{\partial u} \tag{4}$$

and

$$\lambda'(t) = -\frac{\partial H}{\partial x} = -e^{-\rho t} \frac{\partial f(t, x(t), u(t))}{\partial x} - \lambda(t) \frac{\partial g(t, x(t), u(t))}{\partial x}, \lambda(T) = 0. \tag{5}$$

The adjoint variable or co-state $\lambda(t)$ is the marginal value of the capital asset at time t which denotes the increase of the objective function due to marginal increase of the state variable. The reduction of the capital by one unit would reduce the value by approximately $\lambda(t)$, and thus, $\lambda(t)$ is called the *shadow price* of the asset. The Hamiltonian represents the rate of increase of total assets, both accumulated dividends and capital assets [12].

According to Kamien and Schwartz [13] all values are discounted back to time 0, and in particular, the multiplier $\lambda(t)$ gives a marginal valuation of the state variable at t discounted back to time zero. However, it is often convenient to conduct discussion in terms of current values, that is, the values at time t rather than their equivalent at time zero. Thus, we write (3) in the form

$$H = e^{-\rho t}[f(t, x(t), u(t)) + e^{\rho t} \lambda(t) g(t, x(t), u(t))] \tag{6}$$

and define

$$m(t) = e^{\rho t} \lambda(t) \tag{7}$$

as the *current value multiplier* associated with (2). Whereas $\lambda(t)$ gives the marginal value of the state variable at time t, discounted back to time zero, the new current value multiplier $m(t)$ gives the marginal value of the state variable at time t in terms of values at t [13].

Now, let $\mathcal{H}$ be defined as

$$\mathcal{H} = e^{\rho t} H = f(t, x(t), u(t)) + m(t) g(t, x(t), u(t)), \tag{8}$$

then $\mathcal{H}$ is called the *current value* Hamiltonian. From (4) - (7) we have,

$$\frac{\partial \mathcal{H}}{\partial u} = \frac{\partial f(t, x(t), u(t))}{\partial u} + m(t) \frac{\partial g(t, x(t), u(t))}{\partial u}, \tag{9}$$

and

$$m'(t) = \rho m(t) - \frac{\partial f(t, x(t), u(t))}{\partial x} - m(t) \frac{\partial g(t, x(t), u(t))}{\partial x}. \tag{10}$$

(See [5] for details).

Note that (8) and (9) do not contain any discount rate. Thus, we have the following theorem:

Theorem 2.1. [Maximum Principle] *Given the optimal pair* $(x^*(t), u^*(t))$ *for the problem (1)-(2) with the current value Hamiltonian (9), then there exists* $m(t)$ *such that for all* t *in* $[0, T]$,

$$u = u^*(t) \quad \text{maximises } \mathcal{H} \quad \text{for } u \in U, \tag{11}$$

and that whenever $u^*(t)$ *is continuous,*

$$m'(t) = \rho m(t) - \frac{\partial f}{\partial x} - m(t) \frac{\partial g}{\partial x}, \tag{12}$$

with transversality conditions

$$\lambda(T) = e^{-\rho T} m(T) = 0 \quad \text{if } x(T) \text{ is free} \tag{13}$$

or

$$e^{-\rho T} m(T) \geq 0 \quad \text{and} \quad e^{-\rho T} m(T) x(T) = 0 \quad \text{if } x(T) \geq 0. \tag{14}$$

The details of this theorem can be obtained in [13, 15-17]. The maximum principle tell us that the control must maximise the Hamiltonian, and to do so one must be able to find the shadow value that satisfies a certain differential equation. In other words, the maximum principle reduces the optimal control problem to the question of determining the shadow values. The maximum principle can be generalised and be used to a system of differential equations.

2.2 The RVF Epidemic Model

The epidemic model of RVF under this study considers three populations: mosquitoes, livestock, and humans with vertical transmission in mosquitoes considered to be the fraction of the number of births of mosquitoes. Since the study is mainly on optimal control under budget constraint, our main concern will be on handling the infected and susceptible individuals specifically livestock and humans. Therefore, the model will be of SI-type in case of mosquitoes, and SIR-type in case of livestock and humans. The population for mosquitoes consists of susceptible mosquitoes (S_m), and infectious mosquitoes (I_m). The livestock population consists of susceptible livestock (S_l), infectious livestock (I_l), and recovered livestock (R_l). The human population consists of susceptible humans (S_h), infectious humans (I_h) and recovered humans (R_h). The model parameters and their description as they have been used in this work are given in Table 1. The model is a system of nonlinear ordinary differential equations of first order as presented by the following equations. Mosquitoes

$$\frac{dS_m}{dt} = b_m(1-f_m)N_m - \mu_m S_m - \lambda_{lm}\frac{I_l}{N_l}S_m - \lambda_{hm}\frac{I_h}{N_h}S_m, \tag{15a}$$

$$\frac{dI_m}{dt} = b_m f_m N_m + \lambda_{lm}\frac{I_l}{N_l}S_m + \lambda_{hm}\frac{I_h}{N_h}S_m - \mu_m I_m. \tag{15b}$$

Livestock

$$\frac{dS_l}{dt} = b_l N_l - \mu_l S_l - \lambda_{ml}\frac{I_m}{N_m}S_l, \tag{16a}$$

$$\frac{dI_l}{dt} = \lambda_{ml}\frac{I_m}{N_m}S_l - (\mu_l + \phi_l + \gamma_l)I_l, \tag{16b}$$

$$\frac{dR_l}{dt} = \gamma_l I_l - \mu_l R_l. \tag{16c}$$

Humans

$$\frac{dS_h}{dt} = b_h N_h - \mu_h S_h - \lambda_{lh}\frac{I_l}{N_l}S_h - \lambda_{mh}\frac{I_m}{N_m}S_h, \tag{17a}$$

$$\frac{dI_h}{dt} = \lambda_{lh}\frac{I_l}{N_l}S_h + \lambda_{mh}\frac{I_m}{N_m}S_h - (\mu_h + \phi_h + \gamma_h)I_h, \tag{17b}$$

$$\frac{dR_h}{dt} = \gamma_h I_h - \mu_h R_h. \tag{17c}$$

To simplify the model, we consider that each population is of fixed size and we let S_i, I_i be the proportions of the susceptibles and infectives respectively for $i = m, l, h$ (mosquitoes, livestock and humans); and R_i be the proportion of the recovered for $i = l, h$ (livestock and humans). Thus, the new system of equations will be

$$\frac{dS_m}{dt} = b_m(1 - f_m) - \mu_m S_m - \lambda_{lm} I_l S_m - \lambda_{hm} I_h S_m, \tag{18a}$$

$$\frac{dI_m}{dt} = b_m f_m + \lambda_{lm} I_l S_m + \lambda_{hm} I_h S_m - \mu_m I_m \tag{18b}$$

$$\frac{dS_l}{dt} = b_l - \mu_l S_l - \lambda_{ml} I_m S_l, \tag{18c}$$

$$\frac{dI_l}{dt} = \lambda_{ml} I_m S_l - (\mu_l + \phi_l + \gamma_l) I_l, \tag{18d}$$

$$\frac{dR_l}{dt} = \gamma_l I_l - \mu_l R_l \tag{18e}$$

$$\frac{dS_h}{dt} = b_h - \mu_h S_h - \lambda_{lh} I_l S_h - \lambda_{mh} I_m S_h, \tag{18f}$$

$$\frac{dI_h}{dt} = \lambda_{lh} I_l S_h + \lambda_{mh} I_m S_h - (\mu_h + \phi_h + \gamma_h) I_h, \tag{18g}$$

$$\frac{dR_h}{dt} = \gamma_h I_h - \mu_h R_h. \tag{18h}$$

The objective of the optimal problem is to minimize the infection during the outbreak of the epidemic by optimising the current value Hamiltonian for the disease dynamics equations subject to epidemiological and economical constraints. That is, minimise the objective functional

$$J = \int_0^\infty e^{-\rho t}(I_l + I_h)dt, \tag{19}$$

where ρ is the discounting rate.

2.3 The Economic Optimal Control Problem

Suppose that the expenditure on control is subject to a budget constraint $c(I_l + I_h) \leq M$ where c is the cost of intervention strategy per infected individual and M is the expenditure limit (available resource budget). If there are sufficient resources, all infected individuals will be attended. Otherwise, resources are allocated so as to minimise the discounted number of infected individuals in both groups over time. If only the infected individuals are being attended to, then αI_i, $i = l, m$ infected individuals will leave the infectious group in livestock and humans.

Therefore, we have

$$\frac{dI_l}{dt} = \lambda_{ml} I_m S_l - (\mu_l + \phi_l + \gamma_l) I_l - \alpha I_l, \tag{20a}$$

$$\frac{dI_h}{dt} = \lambda_{lh} I_l S_h + \lambda_{mh} I_m S_h - (\mu_h + \phi_h + \gamma_h) I_h - \alpha I_h, \tag{20b}$$

where α is the measure of the rate at which infected individuals are attended by using the available intervention.

Let $\mathcal{D} = \{I_l, I_h \geq 0 \ni I_l + I_h \leq M/c\}$ be the region where there are sufficient resources to attend all the infected individuals. Then all points on $\mathcal{D}$ and inside $\mathcal{D}$ remains permanently within $\mathcal{D}$ and eventually converge to a stable equilibrium. The objective functional in this case is defined by the functional V such that

$$V(I_l, I_h) = \int_0^\infty e^{-\rho t}(I_l + I_h)dt \tag{21}$$

where the integral is evaluated along the path starting from the point $I_l(0) = I_l$ and $I_h(0) = I_h$.

We are interested in the case where there is insufficient resources to attend to all infected individuals; that is, the points outside $\mathcal{D}$ for which $I_l + I_h > M/c$. Suppose that the number of infected individuals being attended in population i equals F_i, $i = l, m$. With the assumption that only infected individuals attended, then $F_i \leq I_i$. The dynamics of the infection outside $\mathcal{D}$ are then given by

$$\frac{dI_l}{dt} = \lambda_{ml} I_m S_l - (\mu_l + \phi_l + \gamma_l) I_l - \alpha F_l, \tag{22a}$$

$$\frac{dI_h}{dt} = \lambda_{lh} I_l S_h + \lambda_{mh} I_m S_h - (\mu_h + \phi_h + \gamma_h) I_h - \alpha F_h. \tag{22b}$$

Under budget constraints, $c(F_l + F_h) \leq M$ and therefore, we choose F_l and F_h so as to minimise the total infection across the populations

$$J = \int_0^\infty e^{-\rho t}(I_l + I_h)dt \tag{23}$$

subject to epidemiological and economical constraints

$$I_i(0) = I_{i,0},\ 0 \leq F_i \leq I_i,\ F_l + F_h = \min(I_l + I_h, M/c),\ i = l, m.$$

Since there are more infected than we can attend to, $c(I_l + I_h) > M$ and hence $F_l + F_h = M/c$ or $F_h = M/c - F_l$. Thus, we have

$$\frac{dI_l}{dt} = \lambda_{ml} I_m S_l - (\mu_l + \phi_l + \gamma_l) I_l - \alpha F_l, \tag{24a}$$

$$\frac{dI_h}{dt} = \lambda_{lh} I_l S_h + \lambda_{mh} I_m S_h - (\mu_h + \phi_h + \gamma_h) I_h - \alpha(M/c - F_l). \tag{24b}$$

Since $\min_u J = -\max_u J$, using equation (8) the current value Hamiltonian of this problem is given by

$$\mathcal{H} = -(I_l + I_h) + m_1 \frac{dS_m}{dt} + m_2 \frac{dI_m}{dt} + m_3 \frac{dS_l}{dt} + m_4 \frac{dI_l}{dt} + m_5 \frac{dS_h}{dt} + m_6 \frac{dI_h}{dt} \tag{25}$$

or

$$\begin{aligned} \mathcal{H} = & -(I_l + I_h) + m_1[b_m(1 - f_m) - \mu_m S_m - \lambda_{lm} I_l S_m - \lambda_{hm} I_h S_m] \\ & + m_2[b_m f_m + \lambda_{lm} I_l S_m + \lambda_{hm} I_h S_m - \mu_m I_m] + m_3[b_l - \mu_l S_l - \lambda_{ml} I_m S_l] \\ & + m_4[\lambda_{ml} I_m S_l - (\mu_l + \phi_l + \gamma_l) I_l - \alpha F_l] \\ & + m_5[b_h - \mu_h S_h - \lambda_{lh} I_l S_h - \lambda_{mh} I_m S_h] \\ & + m_6[\lambda_{lh} I_l S_h + \lambda_{mh} I_m S_h - (\mu_h + \phi_h + \gamma_h) I_h - \alpha(M/c - F_l)]. \end{aligned} \tag{26}$$

where m_i for $i = 1, 2, \cdots, 6$ are costate variables, and that $F_l \geq 0$, $I_l - F_l \geq 0$, $M/c - F_l \geq 0$, $I_h - F_h = F_l + I_h - M/c \geq 0$. The Lagrangian of this problem is given by

$$\mathcal{L} = \mathcal{H} + w_1 F_l + w_2(I_l - F_l) + w_3(M/c - F_l) + w_4(F_l + I_h - M/c) \tag{27}$$

where $w_i \geq 0, \quad i = 1, 2, 3, 4$, are penalties multiplier such that $w_1 F_l = 0$, $w_2(I_l - F_l) = 0$, $w_3(M/c - F_l) = 0$, $w_4(F_l + I_h - M/c) = 0$. The first-order optimality conditions requires that

$$\frac{\partial \mathcal{L}}{\partial F_l} = \alpha(m_6 - m_4) + w_1 - w_2 - w_3 + w_4 = 0 \tag{28}$$

and that F_l (and consequently F_h) to be chosen to minimise the Hamiltonian. Thus,

if $m_6 - m_4 > 0$, then $F_l = \min\{I_l, M/c\}$ and $F_h = M/c - F_l$, and

if $m_6 - m_4 < 0$, then $F_h = \min\{I_h, M/c\}$ and $F_l = M/c - F_h$.

Using Theorem , we have

(29a) $$m_1'(t) = (\rho+\mu_m)m_1 + (m_1 - m_2)(\lambda_{lm}I_l + \lambda_{hm}I_h)$$

(29b) $$m_2'(t) = (\rho+\mu_m)m_2 + (m_3 - m_4)\lambda_{ml}S_l + (m_5 - m_6)\lambda_{mh}S_h$$

(29c) $$m_3'(t) = (\rho+\mu_l)m_3 + (m_3 - m_4)\lambda_{ml}I_m$$

(29d) $$m_4'(t) = 1 + \rho m_4 + (m_1 - m_2)\lambda_{lm}S_m + (m_5 - m_6)\lambda_{lh}S_h + m_4(\mu_l + \phi_l + \gamma_l) + w_2$$

(29e) $$m_5'(t) = (\rho+\mu_h)m_6 + (m_5 - m_6)(\lambda_{lh}I_l + \lambda_{mh}I_m)$$

(29f) $$m_6'(t) = 1 + \rho m_6 + (m_1 - m_2)\lambda_{hm}S_m + m_6(\mu_h + \phi_h + \gamma_h) + w_4.$$

2.4 Interior solution

Suppose there exists a path which satisfies the first-order optimality conditions above. For $w_1 = w_2 = w_3 = w_4 = 0$, there exists an open interval for which $m_4(t) = m_6(t)$. Taking the derivative of $m_6(t) - m_4(t)$ over the open interval, we have,

(30) $$\begin{aligned} m_6'(t) - m_4'(t) &= (m_1 - m_2)(\lambda_{hm} - \lambda_{lm})S_m + (m_6 - m_5)\lambda_{lh}S_h \\ &\quad + m_6(\mu_h + \phi_h + \gamma_h) - m_4(\mu_l + \phi_l + \gamma_l) \\ &= 0. \end{aligned}$$

Since $m_4(t) = m_6(t)$, the equation (30) can also be written as

(31) $$\begin{aligned} &-(m_1 - m_2)(\lambda_{lm} - \lambda_{hm})S_m - (m_5 - m_6)\lambda_{lh}S_h \\ &+ m_6(\mu_h + \phi_h + \gamma_h) - m_4(\mu_l + \phi_l + \gamma_l) = 0. \end{aligned}$$

The economic interpretation is that, the co-state variables are the shadow prices [12-13], where the variables m_i indicates the marginal benefit to the society of increasing one unit of the proportion of susceptible and infected individuals. Since infection is harmful, increasing one unit of infected individuals decreases one unit of susceptible individuals. Thus, the shadow prices $m_2, m_4,$ and m_6 must be negative. Therefore, $m_2, m_4,$ and m_6 represent the amount that the society is willing to invest for control of the RVF epidemic that would result in reducing the number of infected individuals by unit in each population. The shadow prices m_1, m_3 and m_5 must be positive such that $m_1 - m_2 \geq 0$, $m_3 - m_4 \geq 0$, and $m_5 - m_6 \geq 0$.

There are two cases to the solution of the equation (31). First, is the case when $m_1 = m_2$ and $m_4 = m_5 = m_6 = 0$, and the case when $\lambda_{lm} = \lambda_{hm}$ and $m_4 = m_5 = m_6 = 0$. But m_4 and m_6 are negative and satisfies equation (28), therefore, one can not find an interval for which $m_4 = m_6$. The sign of $m_6 - m_4$ will therefore be constant or switch from negative to positive once.

2.5 Optimal solution

The optimal control strategies depend on the effect of the marginal change in the value of $m_6(t) - m_4(t)$. From the discussion on interior solutions, we find that, the sign of $m_6 - m_4$ will either be constant or switch from negative to positive only once. Since the co-state variables are interpreted as shadow prices, it follows that, if increasing the amount of infected individuals in livestock by one unit, would generate more infection in the whole population than an increase of the same in humans, then preference in intervention must be given to livestock and vice versa. Thus, whenever $I_l + I_h > M/c$, the optimal intervention strategy is such that

High transmission strategy: $F_l = \min\{I_l, M/c\}$, $F_h = M/c - F_l$.

Low transmission strategy: $F_h = \min\{I_h, M/c\}$, $F_l = M/c - F_h$.

Switching strategy: Switching high to low strategy with single switch from implementing the high transmission strategy to low transmission strategy.

By high transmission strategy we refer to giving priority to population with the higher transmission rate, and low transmission strategy refer to giving priority to population with the lower transmission rate. Since only human can receive infection from livestock, we assume asymmetric rate of transmission in which livestock is the one mainly driving the epidemic.

3. Results and discussion

In this section we solve the optimal control problem under budget constraints using fourth order Runge-Kutta scheme. Using the parameter values in Table 1, numerical simulations were carried out with state system initial values $S_m(0) = 0.70, I_m(0) = 0.25, S_l(0) = 0.75, I_l(0) = 0.25, R_l(0) = 0.00, S_h(0) = 0.80, I_h(0) = 0.20, R_h(0) = 0.00$ and final time $t_f = 365$. The results of numerical simulations are presented in Figure 1, 2, and 3.

Figure 1 shows the dynamics of infected livestock and human with a given intervention for $\alpha = 0.25, M = 0.35$, and $c = 1$.

TABLE 1. Description of parameters and their values used in numerical simulations

Parameter	Description	Value
$1/b_m$	birth rate of mosquitoes	100
b_h	daily birthrate in humans	0.0015
b_l	daily birthrate in livestock	0.0025
$1/\mu_m$	lifespan of mosquitoes	3 days
$1/\mu_h$	lifespan of humans	40 yrs
$1/\mu_l$	lifespan of livestock	1 yr
ϕ_l	disease induced death rate in livestock	0.025
ϕ_h	disease induced death rate in humans	0.010
$1/\gamma_l$	infectious period in livestock	1 days
$1/\gamma_h$	infectious period in humans	4 days
λ_{ml}	adequate contact rate: mosquito to livestock	0.20
λ_{lm}	adequate contact rate: livestock to mosquito	0.24
λ_{mh}	adequate contact rate: mosquito to humans	0.025
λ_{hm}	adequate contact rate: humans to mosquito	0.0025
λ_{lh}	adequate contact rate: livestock to humans	0.001
f_m	vertical transmission rate in mosquitoes	0.05
ρ	discounting rate	0.1

Figure 2 shows the proportion of treated or attended individuals in livestock and human population. From the graph, we find that initially more resources are placed to human up to $t = 100$ days, when a shift occur to give more weight to livestock. This strategy is the same as giving preference to less infected population before shifting to high infected population. The strategy is also equivalent to giving preference to population with higher proportions of susceptible individuals and then shift to population of lower susceptible individuals after a certain instance of time. The cumulative cost increases for the first 50 days and then attain a constant value for the remainder of the time as in Figure 3.

4. Conclusion

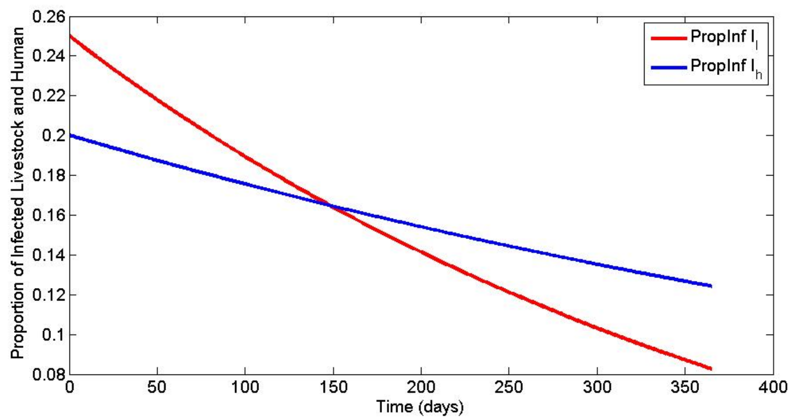

FIGURE 1. Proportions of infected livestock and human with $\alpha = 0.25$, $M = 0.35$, and $c = 1$.

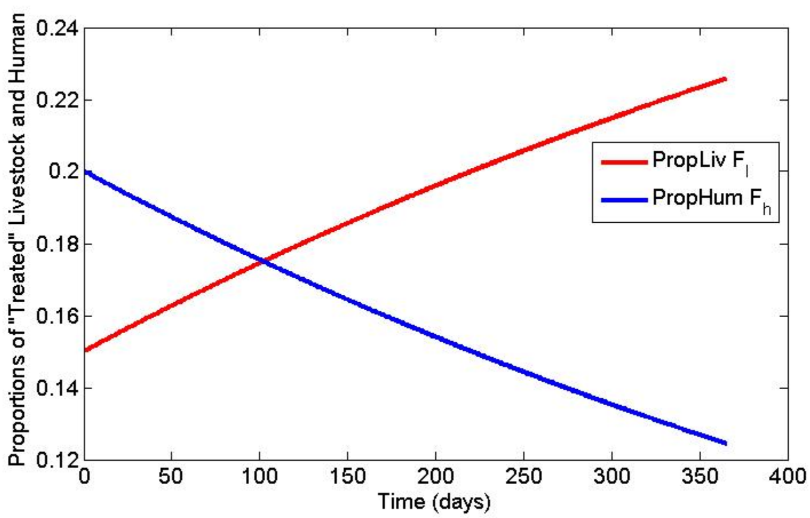

FIGURE 2. Proportional of "treated" livestock and human with $\alpha = 0.25$, $M = 0.35$, and $c = 1$.

An SIR-type optimal control model of RVF under budget constraints have been established to evaluate the potential preference in applying intervention between infected livestock and human. To analyse the model, several assumptions were made including the assumptions that: (1) the resources for control are limited, (2) the expenditure for disease control in livestock and human are drawn from the same budget, and (3) the rates of infection are constant over time. The results shows that resources should be allocated first for interventions focusing on population with lower level of infected individuals (for this case human) and thereafter shift to population with high level of infected individuals (for this case livestock). These results agree with the results established earlier in Rowthorn et al. [4]. Therefore,

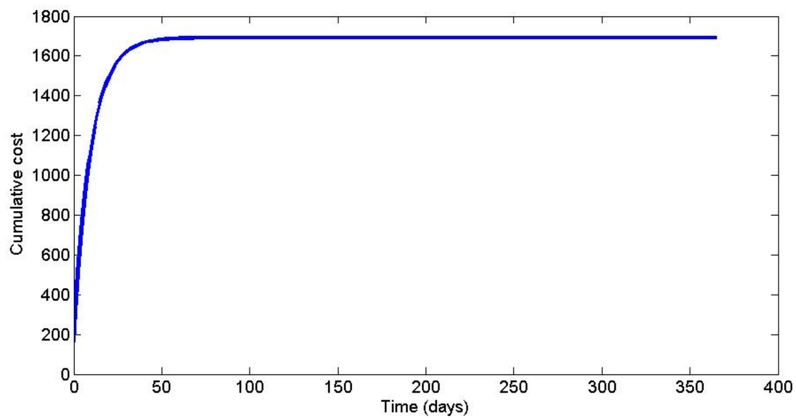

FIGURE 3. cumulative costs of control with $\alpha = 0.25$, $M = 0.35$, and $c = 1$.

the optimal solution is a switching strategy in which priority is given to population with less infected individuals before switching to population with more infected individuals.

Conflict of Interests

The authors declare that there is no conflict of interests.

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REFERENCES

[1] S. C. Mpeshe, H. Heikki, J. M. Tchuenche, A Mathematical model of Rift Valley Fever with human host, Acta Biotheor. 59(2011), 231-250.

[2] S. C. Mpeshe, L. S. Luboobi, Y. Nkansah-Gyekye, Modeling the Impact of Climate Change on the Dynamics of Rift Valley Fever, Comput. Math. Methods Med. 2014 (2014), 1-12.

[3] M. Ndeffo Mbah, C. Gilligan, Resource allocation for epidemic control in metapopulations, PLoS ONE, 6(2011), 1-10.

[4] R. Rowthorn, R. Laxminaryan, C. A. Gilligan, Optimal control of epidemics in metapopulations, J. R. Soc. Interface, 6 (2009), 1135-1144.

[5] M. Ndeffo Mbah, C. Gilligan, Optimization of control strategies of epidemics in heterogeneous population with symmetric and asymmetric transmission, J. Theoret. Bio. 262 (2009), 757-763.

[6] H. Behncke, Optimal control of deterministic epidemics, Optim. Contr. Appl. Meth. 21 (2000), 269-285.

[7] D. Smith, S. Levin, R. Laxminaryan, Strategic interactions in multi-institutional epidemics of antibiotic resistance, Proc. Nat. Acad. Sci. USA, 102 (2005), 3153-3158.

[8] R. Rowthorn, Optimal treatment of disease under budget constraint, In R. Halvorsen and D. Layton (Eds.), Explorations in Environmental and Natural Resource Economics: Essays in Honor of Gadner M. Brown Jr., UK: Edward Elgar (2006), 20-35..

[9] G. Foster, C. Gilligan, Optimizing the control of disease infestations at the landscape scale, Proc. Nat Acad. Sci. USA, 104 (2007), 4984-4989.

[10] M. Ndeffo Mbah, C. Gilligan, Balancing detection and eradication for control of epidemics: Sudden oak death in mixed-species stands, PLoS ONE, 5 (2010), 1-8.

[11] E. Hansen, T. Day, Optimal control of epidemics with limited resources, J. Math. Biol. 62 (2011), 423-451.

[12] F. Brauer, S. Castillo-Shàvez, Mathematical Models in Population Biology and Epidemiology, USA: Springer (2001).

[13] N. Kamien, N. Schwartz, Dynamic Optimisation: The Calculus of Variations and Optimal Control in Economics and Management, New York: Elsevier (1981)

[14] P. Chen, S. Islam, Optimal Control Models in Finance: A New Computational Approach, USA: Springer (2005).

[15] A. Seierstad, K. Sydsæter, Optimal Control Theory with Economic Applications, Netherlands: Elsevier (1987).

[16] K. Sydsæter, P. Hammond, A. Seierstad, A. Strøm, Further Mathematics for Economic Analysis (3rd edition), UK: Prentice Hall (2008).

[17] T. Weber, Optimal Control Theory with Applications in Economics, UK: MIT (2011).